Cylinder under pure compression

Fino test case 006-cylinder-pure-compression

Title	**Cylinder under pure compression**
Tags	elasticity compression
Runnng time	1 sec
See also	003-cube-pure-tension
CAEplex case	`https://caeplex.com/p/ed77e`
Available in	HTML PDF ePub

1 Problem description

A non-dimensional cylinder of radius $1/2$ and height 1 whose base rests on the x-y plane centered at the origin is subject to a nondimensional compressive pressure of 1 unit in the negative z direction acting on the face at $z = 1$ (fig. 1). The cube has a non-dimensional unitary Young modulus $E = 1$ and Poisson's ratio $\nu = 0.3$. To remove rigid-body displacemetns while obtaining pure compressive stress, instead of fixing lower-dimensional entities as in Cube under pure tension, the bottom face is set to have both *tangential* and *radial* symmetry displacement conditions.

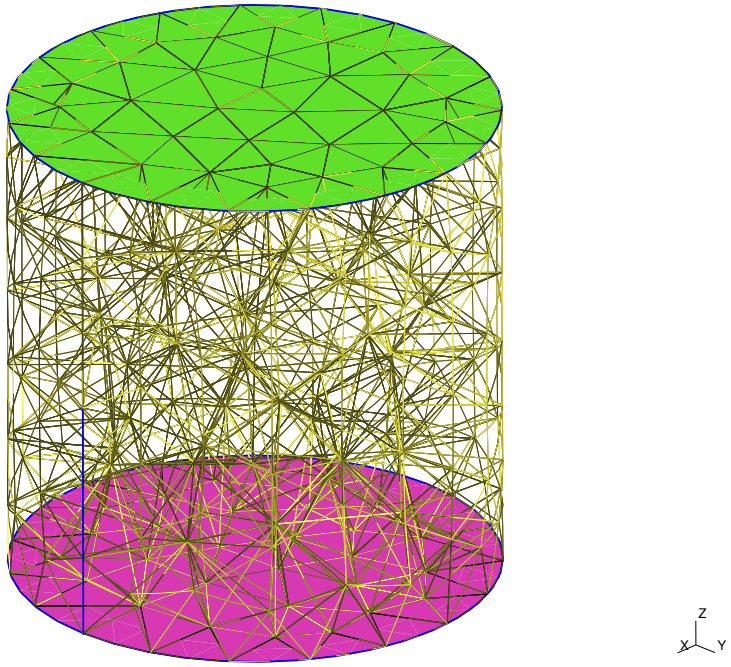


Figure 1: Non-dimensional cylinder and mesh

Once again, as in Cube under pure tension, the displacements and stresses distribution within the cylinder are to be obtained. The stress σ_x is expected to be uniformly equal to one with the other five stress

components (σ_y, σ_z, τ_{xy}, τ_{yz} and τ_{zx}) to be identically equal to zero. The cylinder is expected to shrink in the z direction and to expand in both the x and y directions due to the non-zero Poisson's ratio ν.

The mesh is expected to be fully unstructured and not optimized as to show that the expected results are recovered with the numerical calculation independently of having a mesh composed of wedges and/or hexahedra within an azimuthal array. In this case, the actual area of the discretized surface that holds the external load does depend on the mesh coarseness, with the theoretical value of $\pi/4$ recovered only in the limit of infinite elements. However, the load condition in this case is an uniform pressure and not a force, so the normal stress in the axial direction is expected to be numerically equal to such pressure.

2 Geometry and mesh

The geometry is created and meshed directly from Gmsh. Only two physical surfaces are needed for the boundary conditions, namely "bottom" and "top." The physical volume is needed to get volumetric elements in the resulting `cylinder.msh` file. The mesh is unstructured tetrahedral due to the reasons already explained.

```
SetFactory("OpenCASCADE");

Cylinder(1) = {0, 0, 0, 0, 0, 1, 0.5};

Physical Surface("bottom") = {3};
Physical Surface("top") = {2};
Physical Volume("bulk") = {1};

Mesh.ElementOrder = 2;
```

3 Input file

This time the input file `cylinder.fin` is not annotated yet again it is self-explanatory. It solves the problem and writes a VTK file.

```
MESH FILE_PATH cylinder.msh

E = 1
nu = 0.3

PHYSICAL_GROUP bottom BC tangential radial
PHYSICAL_GROUP top    BC P=1

FINO_STEP

MESH_POST FILE_PATH cylinder.vtk VECTOR u v w sigmax sigmay sigmaz tauxy tauyz tauzx
```

Note the two special boundary conditions `tangential` and `radial`. The first one restricts displacements along the local normal of the surface. The latter forces displacements within the surface plane to be radially distributed around the surface's barycenter.

4 Execution

```
$ gmsh -v 0 -3 cylinder.geo
$ fino cylinder.fin
$
```

5 Results

Figs. 2a, 2b show σ_x and σ_y in the final deformed condition.

5.1 Check

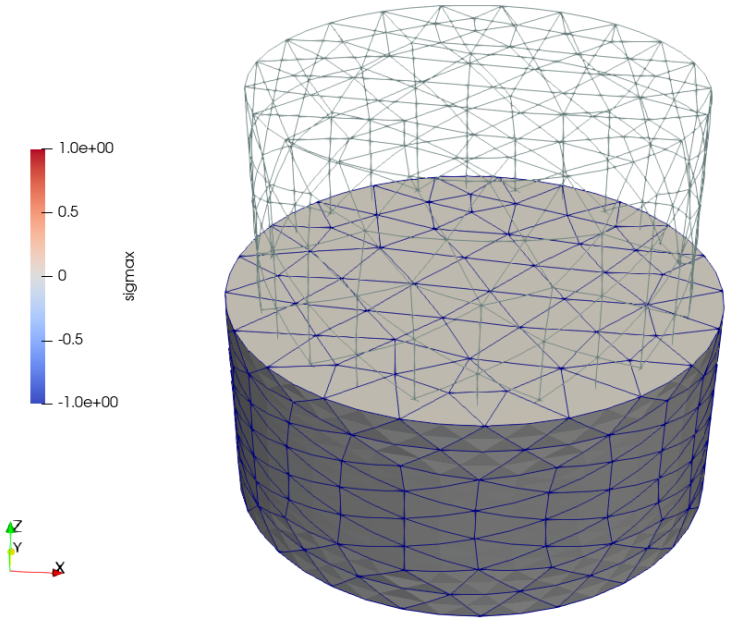
The $\sigma_z(x, y, z)$ distribution is expected to be identically equal to minus one and all the other five stresses are expected to vanish. To verify this is the case, the following input file is built which includes the basic solution, finds the maximum and minimum values of all the six stresses and prints them out in the standard output.

As handy as the `radial` keyword is to simplify the solution of symmetric problems, there are two catches a cognizant user needs to be aware of. On the one hand, this kind of boundary condition is not a traditional essential BC where the value of the unknown (in this case displacements) is set up to the solver's tolerance, but a multi-freedom constrain which is implemented with a penalty method. In the general case, the `tangential` condition is also implemented as a multi-freedom constrain. Yet Fino is smart enough and detects that the outward normal is $(0, 0, -1)$ and then condition is translated into a traditional Dirichlet-type condition $w = 0$.

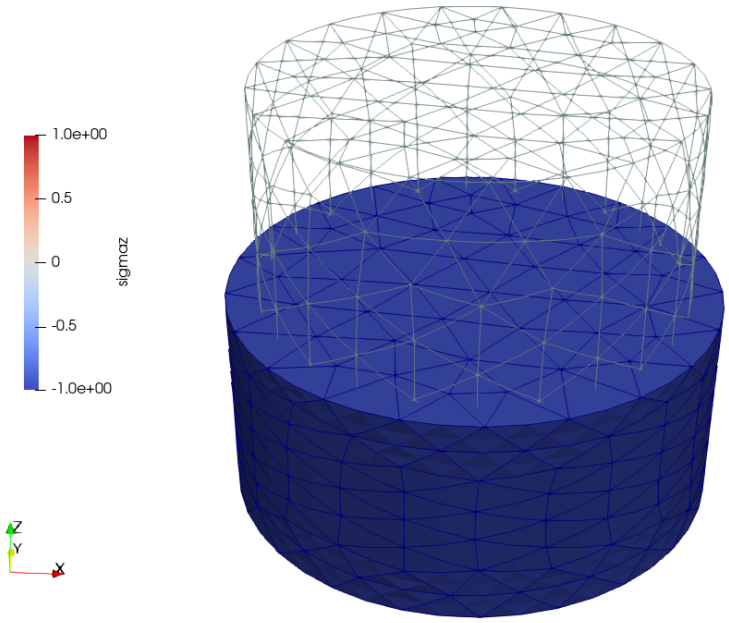
Hence the radial condition is *approximately* set, and the extent to which this setting applies depends on a number of things—including of course the penalty weight. This approximation propagates down to the results, as illustrated in the following check test, which reads both the solver's tolerance (default 10^{-6}) and the penalty weight (default 10^8) from the command line.

```
fino_reltol = $1
fino_penalty_weight = $2

INCLUDE cylinder.fin

# compute min/max stresses
MESH_FIND_MINMAX FUNCTION sigmax MIN sigmax_min MAX sigmax_max
MESH_FIND_MINMAX FUNCTION sigmay MIN sigmay_min MAX sigmay_max
MESH_FIND_MINMAX FUNCTION sigmaz MIN sigmaz_min MAX sigmaz_max
MESH_FIND_MINMAX FUNCTION tauxy  MIN tauxy_min  MAX tauxy_max
MESH_FIND_MINMAX FUNCTION tauyz  MIN tauyz_min  MAX tauyz_max
MESH_FIND_MINMAX FUNCTION tauzx  MIN tauzx_min  MAX tauzx_max

PRINT "stress\tmin\t\tmax"
PRINT SEP "\t" "% +5e" "sigmax" sigmax_min sigmax_max
PRINT SEP "\t" "% .5e" "sigmay" sigmay_min sigmay_max
PRINT SEP "\t" "% .5e" "sigmaz" sigmaz_min sigmaz_max
PRINT SEP "\t" "% .5e" "tauxy"  tauxy_min  tauxy_max
PRINT SEP "\t" "% .5e" "tauyz"  tauyz_min  tauyz_max
PRINT SEP "\t" "% .5e" "tauzx"  tauzx_min  tauzx_max
```

```
$ fino check.fin 1e-3 1e4
```



(a) σ_x



(b) σ_z

Figure 2: Stresses over the displaced grid and comparison with the original cube.

```
stress  min             max
sigmax  -1.861889e-03   +2.982465e-03
sigmay  -2.07812e-03     3.01866e-03
sigmaz  -1.00348e+00    -9.96333e-01
tauxy   -1.34599e-03     1.16549e-03
tauyz   -1.48900e-03     1.04906e-03
tauzx   -1.34727e-03     1.66753e-03
$ fino check.fin 1e-3 1e12
stress  min             max
sigmax  -3.368149e-03   +1.916099e-03
sigmay  -2.82702e-03     3.11235e-03
sigmaz  -1.00313e+00    -9.96947e-01
tauxy   -8.14560e-04     7.82537e-04
tauyz   -1.44005e-03     1.47797e-03
tauzx   -1.70246e-03     1.29679e-03
$ fino check.fin 1e-9 1e4
stress  min             max
sigmax  -6.090733e-04   +4.732937e-04
sigmay  -7.18812e-04     5.77535e-04
sigmaz  -1.00135e+00    -9.97135e-01
tauxy   -6.64507e-04     6.27475e-04
tauyz   -1.08775e-03     1.17853e-03
tauzx   -1.13649e-03     1.02294e-03
$ fino check.fin 1e-9 1e12
stress  min             max
sigmax  -3.826204e-03   +1.897524e-03
sigmay  -2.78529e-03     2.49913e-03
sigmaz  -1.00174e+00    -9.97135e-01
tauxy   -7.08823e-04     1.06614e-03
tauyz   -1.08773e-03     1.17850e-03
tauzx   -1.65213e-03     1.02282e-03
$
```

On the other hand, the imposition of the radial symmetry does depend on the mesh size (take a minute and think about it!). In effect, the input file `check-fine.fin` solves the same proble as `check.fin` with a finer mesh. For the default values of the tolerance and the weight, we get

```
$ fino check.fin 1e-6 1e8
stress  min             max
sigmax  -6.090465e-04   +4.738302e-04
sigmay  -7.17874e-04     5.77485e-04
sigmaz  -1.00135e+00    -9.97133e-01
tauxy   -6.65185e-04     6.28188e-04
tauyz   -1.08804e-03     1.17926e-03
tauzx   -1.13719e-03     1.02356e-03
$ fino check-fine.fin 1e-6 1e8
stress  min             max
sigmax  -1.360267e-04   +1.208525e-04
sigmay  -1.17141e-04     1.03318e-04
sigmaz  -1.00021e+00    -9.99558e-01
tauxy   -1.01711e-04     1.00290e-04
tauyz   -2.23632e-04     2.59015e-04
tauzx   -2.37351e-04     2.68281e-04
$
```

The conclusion is that this kind of "handy" constrain limits the precision of the stresses both through the weight of the penalty method and through the mesh size, although it is still overall good ($\sim \pm 1\%$) for a rather coarse mesh. Recall that in Cube under pure tension the results only depended on the precision of the linear solver. Once again, if we passed the `--mumps` option, the results would not depend on the first argument but they would still depend on the second one.