Heat conduction

Contents

1 Thermal slabs

1.1 One-dimensional linear

Solve heat conduction on the slab $x \in [0:1]$ with boundary conditions

$$\begin{cases} T(0) = 0 & \text{(left)} \\ T(1) = 1 & \text{(right)} \end{cases}$$

and uniform conductivity. Compute $T\left(\frac{1}{2}\right)$.

Please note that:

- The input written in a self-evident English-like dialect
  - Syntactic sugared plain-text ASCII file
  - Simple problems (like this one) need simple inputs
  - FeenoX follows the UNIX rule of simplicity
- Output is 100% user-defined
  - No `PRINT` no output
  - Feenox follows the UNIX rule of silence
- There is no node at $x = 1/2 = 0.5$!
  - FeenoX knows how to interpolate
- Mesh separated from problem
  - The geometry comes from a Git-friendly `.geo`

```
Point(1) = {0, 0, 0};          // geometry:
Point(2) = {1, 0, 0};          // two points
Line(1) = {1, 2};              // and a line connecting them!

Physical Point("left") = {1};  // groups for BCs and materials
Physical Point("right") = {2};
Physical Line("bulk") = {1};   // needed due to how Gmsh works

Mesh.MeshSizeMax = 1/3;        // mesh size, three line elements
Mesh.MeshSizeMin = Mesh.MeshSizeMax;
```

  - UNIX rule of composition
  - The actual input file is a Git-friendly `.fee`

```
PROBLEM thermal 1D     # tell FeenoX what we want to solve
READ_MESH slab.msh     # read mesh in Gmsh's v4.1 format
k = 1                  # set uniform conductivity
BC left  T=0           # set fixed temperatures as BCs
BC right T=1           # "left" and "right" are defined in the mesh
SOLVE_PROBLEM          # we are ready to solve the problem
PRINT T(1/2)           # ask for the temperature at x=1/2
```

```
$ gmsh -1 slab.geo
[...]
Info    : 4 nodes 5 elements
Info    : Writing 'slab.msh'...
```

```
[...]
$ feenox thermal-1d-dirichlet-uniform-k.fee
0.5
$
```

2 Non-dimensional transient heat conduction on a cylinder

Let us solve a dimensionless transient problem over a cylinder. Conductivity and heat capacity are unity. Initial condition is a linear temperature profile along the x axis:

$$T(x, y, z, 0) = x$$

The base of the cylinder has a prescribed time and space-dependent temperature

$$T(0, y, z, t) = \sin(2\pi \cdot t) \cdot \sin(2\pi \cdot y)$$

The other faces have a convection conditions with (non-dimensional) heat transfer coefficient $h = 0.1$ and $T_{\text{ref}} = 1$.

```
PROBLEM thermal 3D
READ_MESH cylinder.msh

end_time = 2  # final time [ non-dimensional units ]
# the time step is automatically computed

# initial condition (if not given, stead-state is computed)
T_0(x,y,z) = x

# dimensionless uniform and constant material properties
k = 1
kappa = 1

# BCs
BC hot   T=sin(2*pi*t)*sin(2*pi*y)
BC cool  h=0.1  Tref=1

SOLVE_PROBLEM

# print the temperature at the center of the base vs time
PRINT %e t T(0,0,0) T(0.5,0,0) T(1,0,0)

WRITE_MESH temp-cylinder.msh T

IF done
 PRINT "\# open temp-anim-cylinder.geo in Gmsh to create a quick rough video"
 PRINT "\# run  temp-anim-cylinder.py  to get a nicer and smoother video"
ENDIF
```

```
$ gmsh -3 cylinder.geo
[...]
Info    : Done optimizing mesh (Wall 0.624941s, CPU 0.624932s)
```

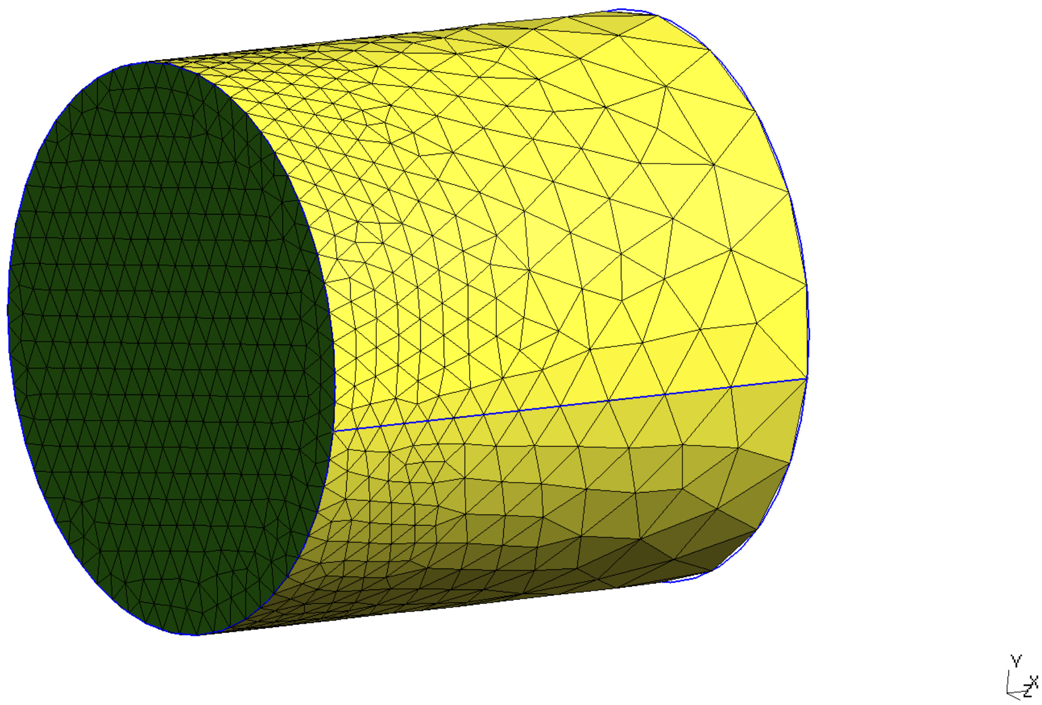

Figure 1: Locally-refined cylinder for a transient thermal problem.

```
Info    : 1986 nodes 10705 elements
Info    : Writing 'cylinder.msh'...
Info    : Done writing 'cylinder.msh'
Info    : Stopped on Fri Dec 24 10:35:32 2021 (From start: Wall 0.800542s, CPU 0.896698s)
$ feenox temp-cylinder-tran.fee
0.000000e+00    0.000000e+00    5.000000e-01    1.000000e+00
1.451938e-04    4.406425e-07    5.000094e-01    9.960851e-01
3.016938e-04    9.155974e-07    5.000171e-01    9.921274e-01
5.566768e-04    1.689432e-06    5.000251e-01    9.862244e-01
8.565589e-04    2.599523e-06    5.000292e-01    9.800113e-01
1.245867e-03    3.780993e-06    5.000280e-01    9.728705e-01
1.780756e-03    5.404230e-06    5.000176e-01    9.643259e-01
2.492280e-03    7.563410e-06    4.999932e-01    9.545723e-01
3.428621e-03    1.040457e-05    4.999538e-01    9.436480e-01
[...]
1.978669e+00    -6.454358e-05   1.500891e-01    2.286112e-01
1.989334e+00    -3.234439e-05   1.500723e-01    2.285660e-01
2.000000e+00    1.001730e-14    1.500572e-01    2.285223e-01
# open temp-anim-cylinder.geo in Gmsh to create a quick rough video
# run  temp-anim-cylinder.py  to get a nicer and smoother video
$ python3 temp-anim-cylinder.py
Info    : Reading 'temp-cylinder.msh'...
Info    : 1986 nodes
Info    : 10612 elements
Info    : Done reading 'temp-cylinder.msh'
0 1 0.0
0.01 12 0.8208905327853042
0.02 15 0.8187351216040447
0.03 17 0.7902629708599855
[...]
Info    : Writing 'temp-cylinder-smooth-198.png'...
Info    : Done writing 'temp-cylinder-smooth-198.png'
199
Info    : Writing 'temp-cylinder-smooth-199.png'...
Info    : Done writing 'temp-cylinder-smooth-199.png'
all frames dumped, now run
ffmpeg -framerate 20 -f image2 -i temp-cylinder-smooth-%03d.png temp-cylinder-smooth.mp4
to get a video
$ ffmpeg -y -f image2 -i temp-cylinder-smooth-%03d.png  -framerate 20 -pix_fmt yuv420p -c:v libx264 -filter: ↩
    v crop='floor(in_w/2)*2:floor(in_h/2)*2'  temp-cylinder-smooth.mp4
[...]
$
```

3 Non-dimensional transient heat conduction with time-dependent properties

Say we have two cubes of non-dimensional size $1 \times 1 \times 1$, one made with a material with unitary properties and the other one whose properties depend explicitly ony time. We glue the two cubes together, fix one side of the unitary material to a fixed zero temperature and set a ramp of temperature between zero and one at the opposite end of the material with time-varying properties.

This example illustrates how to

1. assign different material properties to different volumes
2. give time-dependent material properties and boundary conditions
3. plot temperatures as function of time at arbitrary locations on space

```
PROBLEM thermal 3D
READ_MESH two-cubes.msh

end_time = 50
# initial condition (if not given, stead-state is computed)
# T_0(x,y,z) = 0

# dimensionless uniform and constant material properties
k_left = 0.1+0.9*heaviside(t-20,20)
rho_left = 2-heaviside(t-20,20)
cp_left = 2-heaviside(t-20,20)

# dimensionless uniform and constant material properties
k_right = 1
rho_right = 1
cp_right = 1

# BCs
BC zero  T=0
BC ramp  T=limit(t,0,1)
BC side  q=0

SOLVE_PROBLEM

PRINT t T(0,0,0) T(0.5,0,0) T(1,0,0) T(1.5,0,0) T(2,0,0)
```

```
$ gmsh -3 two-cubes.geo
[...]
$ feenox two-cubes-thermal.fee > two-cubes-thermal.dat
$
```

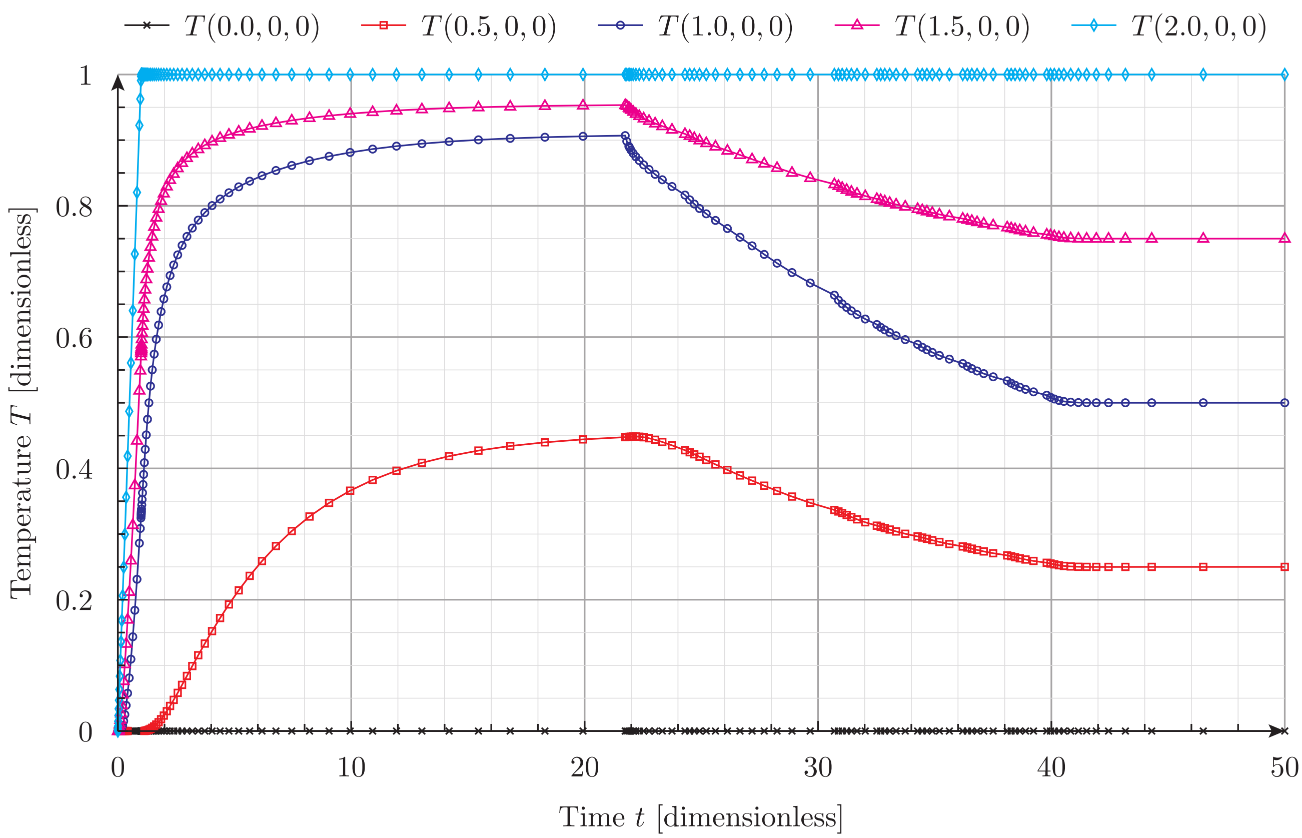

Figure 2: Temporal evolution of temperatures at three locations