Ordinary Differential Equations & Differential-Algebraic Equations

Contents

1 Lorenz' attractor—the one with the butterfly

Solve

$$\begin{cases} \dot{x} &= \sigma \cdot (y - x) \\ \dot{y} &= x \cdot (r - z) - y \\ \dot{z} &= xy - bz \end{cases}$$

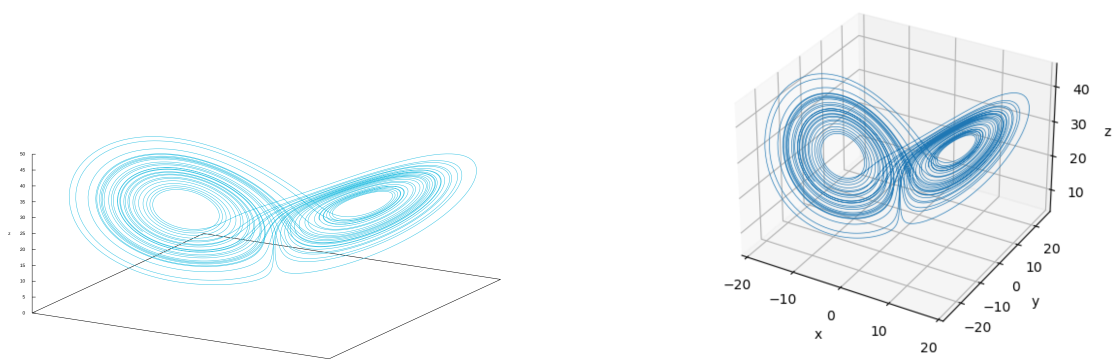
for $0 < t < 40$ with initial conditions

$$\begin{cases} x(0) = -11 \\ y(0) = -16 \\ z(0) = 22.5 \end{cases}$$

and $\sigma = 10$, $r = 28$ and $b = 8/3$, which are the classical parameters that generate the butterfly as presented by Edward Lorenz back in his seminal 1963 paper Deterministic non-periodic flow. This example's input file ressembles the parameters, inital conditions and differential equations of the problem as naturally as possible with an ASCII file.

```
PHASE_SPACE x y z      # Lorenz 'attractors phase space is x-y-z
end_time = 40          # we go from t=0 to 40 non-dimensional units

sigma = 10             # the original parameters from the 1963 paper
r = 28
b = 8/3

x_0 = -11              # initial conditions
y_0 = -16
z_0 = 22.5

# the dynamical system's equations written as naturally as possible
x_dot = sigma*(y - x)
y_dot = x*(r - z) - y
z_dot = x*y - b*z

PRINT t x y z         # four-column plain-ASCII output
```

```
$ feenox lorenz.fee > lorenz.dat
$ gnuplot lorenz.gp
$ python3 lorenz.py
$ sh lorenz2x3d.sh < lorenz.dat > lorenz.html
```



(a) Gnuplot (b) Matplotlib

Figure 1: The Lorenz attractor computed with FeenoX plotted with two different tools