One and two dimensional diffusion kinetics benchmarks

Milonga capabilities to solve time dependent neutron diffusion problems

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Abstract

One and two dimension time dependent benchmarks were solved with a milonga branch. These benchmarks are useful to test the feasibility of adding this feature to milonga. Only data and main results are reported because milonga is open source, so all the information is available on the internet. All the results matched the references.

Revision history

Rev.	Date	Author	
A	6-Sep-2017	gtheler	First issue

Contents

1 Introduction

Milonga [1] is a is a free core-level neutronic code that solves the steady-state multigroup neutron transport equation (either using the diffusion approximation or the discrete ordinates S_N method) over unstructured grids (although simple structured grids can also be used) using either a finite-volumes or a finite-elements discretization scheme. It works on top of the wasora [2] framework, which provides means to parse and understand a high-level plain-text input file containing algebraic expressions, data for function interpolation, differential equations and output instructions amongst other facilities. Therefore, any mathematical computation which can be done by wasora—i.e. parametric calculations, multidimensional optimization, function interpolation and integration, etc.—can be combined with the facilities that milonga provides to solve the neutron diffusion equation.

Calculating fast transients, in which the flux's time derivative cannot be disregarded, is a new feature which is being studied and developed to be added to milonga and, so far, it is a different branch. A set of benchmarks were solved:

ARGONNE CODE CENTER: BENCHMARK PROBLEM BOOK. Identification 6: Infinite Slab Reactor Model [3].

ARGONNE CODE CENTER: BENCHMARK PROBLEM BOOK. Identification 14-A1: Super Prompt-Critical Transient; Two-dimensional Neutron Diffusion Problem, with Adiabatic Heatup and Doppler Feedback in Thermal Reactor [4, 5].

Twigl [5].

In order to solve transients, the delayed neutrons precursors were solved together with the flux in the same equation system. It means they were not added as en external source and there is no need to calculate coupling coefficients.

It is implemented in the finite elements scheme and it is a future work to implement it in finite volumes scheme.

The initial condition is calculated from the critical condition, which is got by dividing the fission cross sections by k_{eff}, and the initial precursor concentrations are in equilibrium with the initial critical flux distribution.

2 Infinite Slab Reactor Model

This benchmark is a set of four ones. Their difference is in the perturbation.

The geometry is one dimensional (Figure 1). The reactor consists of three zones with the data shown in Table 1, the two ones which have a zero boundary condition have the same data at time equal to zero.

The delayed neutron data data is shown in the Table 2.

There are not data about the energy per fission, so it is considered equal to the fission cross section.

There are not data about the delayed neutron emission spectrum, so they are assumed to be equal to the fission spectrum.

Second order elements are used.

The initial k_{eff} is the same for all the cases: $k_{eff} = 0.9015186$ while the reference k_{eff} is $k_{eff} = 0.9015507$

The initial power fraction normalized to 1 is shown in the Table 3 and it is the same for all the cases.

The initial power fraction is not symmetrical (Table 3) because the solver gives a non symmetrical solution (Figure 4); but it is a small difference.

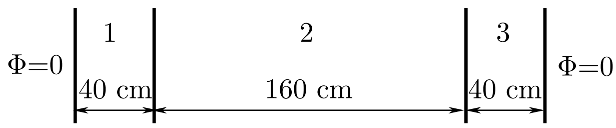

Figure 1: Geometry configuration

Constant	Region 1 and 3	Region 2
D^1 [cm]	1.5	1
D^2 [cm]	0.5	0.5
Σ_a^1 [cm^{-1}][a]	0.026	0.02
Σ_a^2 [cm^{-1}][a]	0.18	0.08
$\nu\Sigma_f^1$ [cm^{-1}]	0.01	0.005
$\nu\Sigma_f^2$ [cm^{-1}]	0.2	0.099
$\Sigma^{1\to2}$ [cm^{-1}]	0.015	0.01
χ^1 [-]	1	1
χ^2 [-]	0	0
v^1 [cm/s]	10^7	10^7
v^2 [cm/s]	3 10^5	3 10^5

[a] Total removal cross section, including Σ_C, Σ_f, and $\Sigma^{1\to2}$.

Table 1: Initial two groups constants

Type	Effective Delay Fraction	Decay Constant [s^{-1}]
1	0.00025	0.0124
2	0.00164	0.0305
3	0.00147	0.111
4	0.00296	0.301
5	0.00086	1.14
6	0.00032	3.01

Table 2: Delayed neutron parameters

Region	Power [-] Milonga	Power [-] Reference
1	0.27881	0.27895
2	0.44242	0.44209
3	0.27884	0.27895

Table 3: Initial power fractions

2.1 Subcritical Transient, 1D 2-groups Neutron Diffusion Problem in Thermal Reactor

The initiating perturbation is that Σ_a^2 in region 1 is linearly increased by 3% in 1 second.

The mesh is uniform with Δx =2cm to compare with [3].

The time step is 0.1 seconds and it is solved with the backwards Euler method.

Results:

The thermal flux at 0, 1 and 2 second is shown in the Figure 2.

The fast flux at 0, 1 and 2 second is shown in the Figure 3.

The power at 0, 1 and 2 second is shown in the Figure 4.

The total power relative to the initial total power is shown in the Table 4.

The power fractions relative to the initial power fractions is shown in the Table 5.

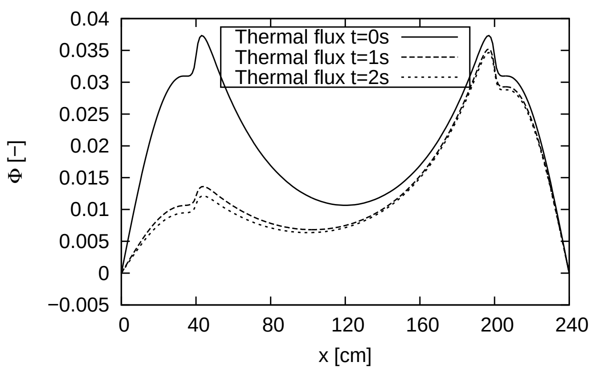

Figure 2: Thermal flux

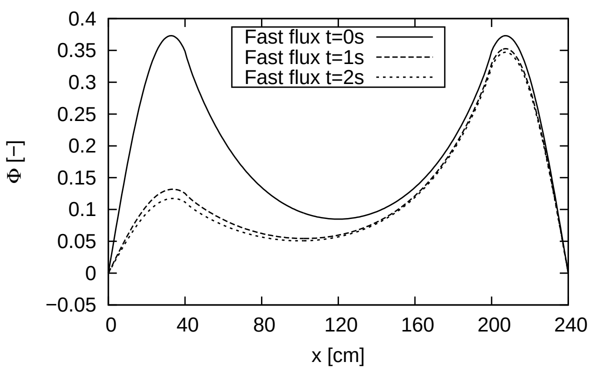

Figure 3: Fast flux

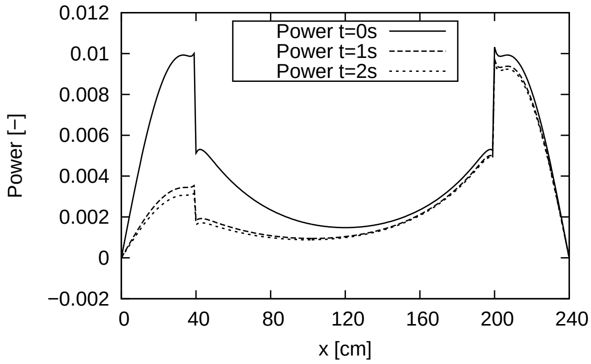

Figure 4: Power

Time [s]	Power [-]	
	Milonga	Reference
0	1	1
0.1	0.9290	0.9298
0.2	0.8720	0.8732
0.5	0.7582	0.7596
1	0.6577	0.6588
1.5	0.6425	0.6432
2	0.6301	0.6306

Table 4: Total Power

	Region					
	Milonga			Reference		
Time [s]	1	2	3	1	2	3
0	1	1	1	1	1	1
0.1	0.8607	0.9331	0.9907	0.8621	0.9339	0.9910
0.2	0.7499	0.8792	0.9826	0.7520	0.8804	0.9830
0.5	0.5313	0.7709	0.9649	0.5336	0.7724	0.9655
1	0.3437	0.6742	0.9456	0.3452	0.6753	0.9462
1.5	0.3226	0.6581	0.9377	0.3235	0.6587	0.9381
2	0.3058	0.6449	0.9307	0.3066	0.6455	0.9311

Table 5: Power fractions

2.2 Delayed Super-critical Transient, 1D 2-groups Neutron Diffusion Problem in Thermal Reactor

The initiating perturbation is that Σ_a^2 in region 1 is linearly decreased by 1% in 1 second.

The mesh is uniform with Δx =2cm to compare with [3].

The time step is 0.05 seconds and it is solved with the Crank-Nicolson method.

Results:

The thermal flux at 0, 2 and 4 second is shown in the Figure 5.

The fast flux at 0, 2 and 4 second is shown in the Figure 6.

The power at 0, 2 and 4 second is shown in the Figure 7.

The total power relative to the initial total power is shown in the Table 6.

The power fractions relative to the initial power fractions is shown in the Table 7.

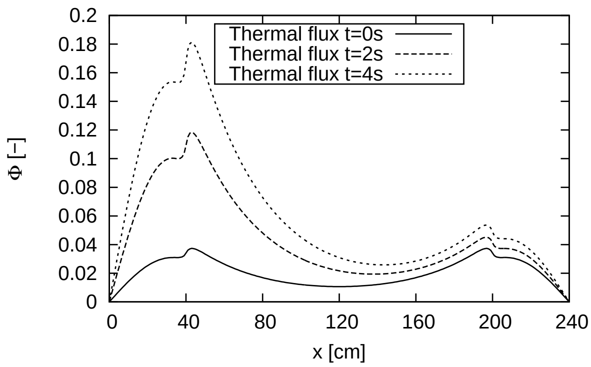

Figure 5: Thermal flux

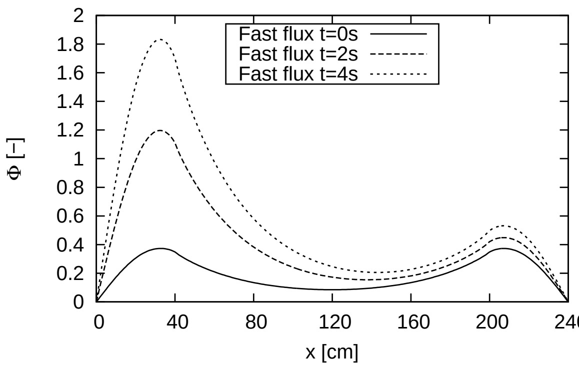

Figure 6: Fast flux

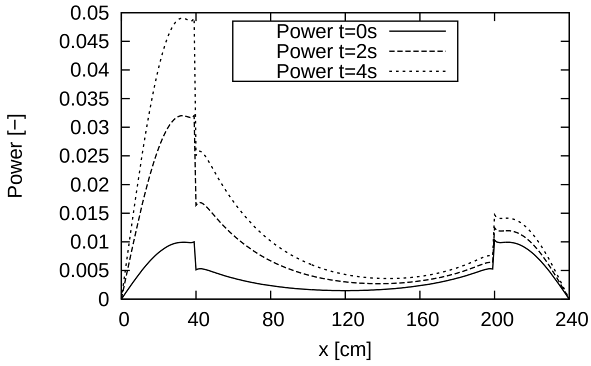

Figure 7: Power

Time [s]	Power [-]	
	Milonga	Reference
0	1	1
0.1	1.0326	1.028
0.2	1.0695	1.062
0.5	1.2194	1.205
1	1.7864	1.740
1.5	1.9701	1.959
2	2.1771	2.165
3	2.6192	2.605
4	3.1246	3.107

Table 6: Total Power

	Region					
	Milonga			Reference		
Time [s]	1	2	3	1	2	3
0	1	1	1	1	1	1
0.1	1.0634	1.0310	1.0046	1.056	1.027	1.004
0.2	1.1350	1.0658	1.0096	-	-	-
0.5	1.4264	1.2078	1.0308	1.399	1.193	1.028
1	2.5248	1.7439	1.1138	2.435	1.701	1.107
1.5	2.8600	1.9216	1.1553	-	-	-
2	3.2379	2.1236	1.2012	3.215	2.113	1.119
3	4.0426	2.5524	1.3019	4.016	2.539	1.298
4	4.9589	3.0426	1.4205	4.927	3.026	1.416

Table 7: Power fractions

2.3 Prompt Super-critical Transient, 1D 2-groups Neutron Diffusion Problem in Thermal Reactor

The initiating perturbation is that Σ_a^2 in region 1 is linearly decreased by 5% in 0.01 second.

The mesh is uniform with Δx =2cm to compare with [3].

The time step is 10^{-5} seconds and it is solved with the Crank-Nicolson method.

Results:

The thermal flux at 0, 0.01 and 0.02 second is shown in the Figure 8.

The fast flux at 0, 0.01 and 0.02 second is shown in the Figure 9.

The power at 0, 0.01 and 0.02 second is shown in the Figure 10.

The total power relative to the initial total power is shown in the Table 8.

The power fractions relative to the initial power fractions is shown in the Table 9.

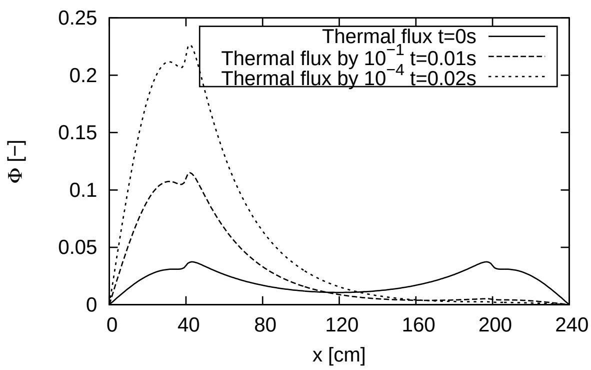

Figure 8: Thermal flux

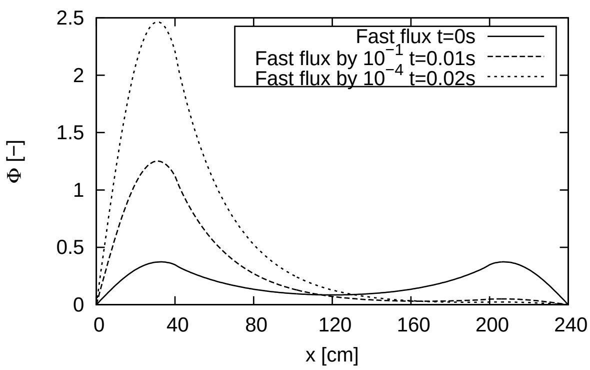

Figure 9: Fast flux

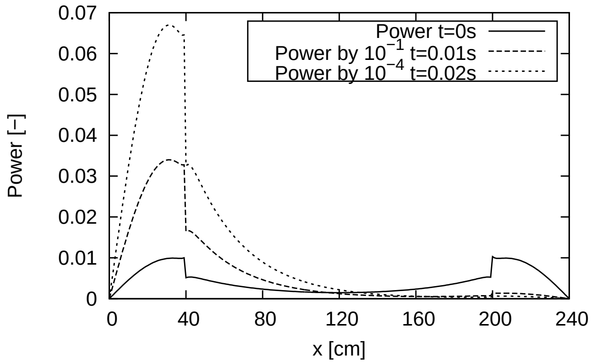

Figure 10: Power

Time [s]	Power [-]	
	Milonga	Reference
0	1	1
0.001	1.0224	1.022
0.005	1.6594	1.659
0.01	15.62	15.65
0.012	69.90	70.19
0.015	674.8	680.3
0.018	6531	6611
0.02	29669	30110

Table 8: Total Power

Time [s]	Region					
	Milonga			Reference		
	1	2	3	1	2	3
0	1	1	1	1	1	1
0.001	1.058	1.014	1	1.058	1.014	1
0.005	2.485	1.543	1.017	2.484	1.544	1.017
0.01	34.77	12.56	1.341	34.81	12.58	1.342
0.012	160.3	55.45	2.396	-	-	-
0.015	1558	534.4	14.75	1570	538.8	14.85
0.018	15086	5172	134.8	-	-	-
0.02	68532	23496	609.1	69540	23850	617.9

Table 9: Power fractions

2.4 Prompt Super-critical Transient, 1D 2-groups Neutron Diffusion Problem in Thermal Reactor with Modified Neutron Velocities

The initiating perturbation is that Σ_a^2 in region 1 is linearly decreased by 5% in 0.01 second.

The mesh is uniform with Δx =2cm to compare with [3].

The time step is 10^{-6} seconds and it is solved with the Crank-Nicolson method.

The fast group velocity is 10^9 cm/s and the thermal velocity is 3 10^7 cm/s.

Results:

The thermal flux at 0, 0.003 and 0.005 second is shown in the Figure 11.

The fast flux at 0, 0.003 and 0.005 second is shown in the Figure 12.

The power at 0, 0.003 and 0.005 second is shown in the Figure 13.

The total power relative to the initial total power is shown in the Table 10.

The power fractions relative to the initial power fractions is shown in the Table 11.

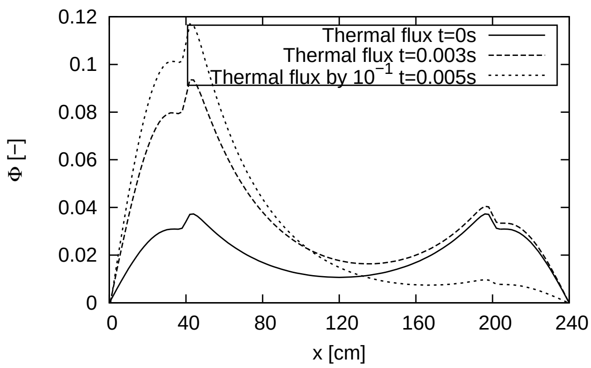

Figure 11: Thermal flux

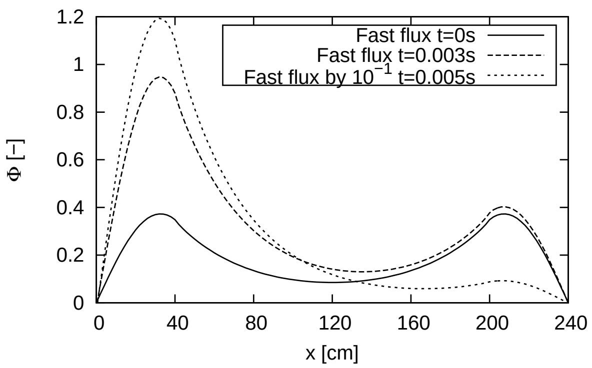

Figure 12: Fast flux

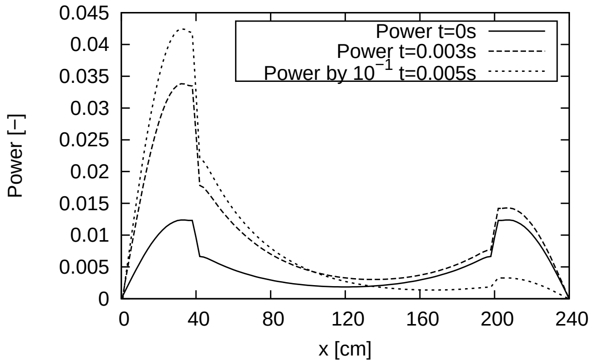

Figure 13: Power

Time [s]	Power [-]	
	Milonga	Reference
0	1	1
0.001	1.178	1.178
0.002	1.558	1.558
0.003	2.796	2.797
0.0035	5.279	5.284
0.004	20.66	20.72
0.0045	467.2	472.0
0.005	150268	153700

Table 10: Total Power

	Region					
	Milonga			Reference		
Time [s]	1	2	3	1	2	3
0	1	1	1	1	1	1
0.001	1.351	1.167	1.022	1.351	1.167	1.022
0.002	2.101	1.524	1.070	2.101	1.525	1.070
0.003	4.546	2.685	1.222	4.547	2.686	1.222
0.0035	9.473	5.007	1.516	-	-	-
0.004	40.24	19.31	3.216	40.36	19.37	3.224
0.0045	946.7	430.3	46.65	-	-	-
0.005	310126	136409	12427	317100	139500	12710

Table 11: Power fractions

2.5 Conclusion

The comparisons with [3] show that milonga agrees with the reference.

3 Two dimensional BWR transient

This benchmark is the 2D case of the LRA BWR Kinetics Problem.

The identification of this benchmark is Super Prompt-Critical Transient; Two-dimensional Neutron Diffusion Problem, with Adiabatic Heatup and Doppler Feedback in Thermal Reactor [4, 5].

This benchmark simulates four control rod ejection because it happens in a quarter of core.

It is a two-dimensional (xy), two groups diffusion theory.

Two delayed neutron precursor groups with zero flux boundary conditions on external surfaces, reflection conditions at symmetry boundaries, and steady state initial conditions, all the fission neutrons appear in the fast flux.

The equations to be solved are:

$$\begin{aligned}
\nabla D_1(\mathbf{x},t)\nabla\Phi_1(\mathbf{x},t) - \left[\Sigma a_1(\mathbf{x},t) + \Sigma_{1\to 2}(\mathbf{x},t)\right]\Phi_1(\mathbf{x},t) + \\
\nu(1-\beta)\left[\Sigma f_1(\mathbf{x},t)\Phi_1(\mathbf{x},t) + \Sigma f_2\Phi_2(\mathbf{x},t)\right] = \frac{1}{v_1}\frac{\partial\Phi_1(\mathbf{x},t)}{\partial t} \\
\nabla D_2(\mathbf{x},t)\nabla\Phi_2(\mathbf{x},t) - \Sigma a_2(\mathbf{x},t) + \Sigma_{1\to 2}(\mathbf{x},t)\Phi_1(\mathbf{x},t) = \frac{1}{v_2}\frac{\partial\Phi_2(\mathbf{x},t)}{\partial t} \\
\nu\beta_i\left[\Sigma f_1(\mathbf{x},t)\Phi_1(\mathbf{x},t) + \Sigma f_2(\mathbf{x},t)\Phi_2(\mathbf{x},t)\right] - \lambda_i C_i(\mathbf{x},t) = \frac{\partial C_i}{\partial t},\ i = 1,2.
\end{aligned}$$

Adiabatic heatup:

$$\alpha\left[\Sigma f_1(\mathbf{x},t)\Phi_1(\mathbf{x},t) + \Sigma f_2(\mathbf{x},t)\Phi_2(\mathbf{x},t)\right] = \frac{\partial T(\mathbf{x},t)}{\partial t}$$

Doppler feedback:

$$\Sigma a_1(\mathbf{x},t) = \Sigma a_1(\mathbf{x},0)\left[1+\gamma\left(\sqrt{T(\mathbf{x},t)}-\sqrt{T_0}\right)\right]$$

Power:

$$P(\mathbf{x},t) = \epsilon\left[\Sigma f_1(\mathbf{x},t)\Phi_1(\mathbf{x},t)+\Sigma f_2(\mathbf{x},t)\Phi_2(\mathbf{x},t)\right]$$

The Figure 14 shows the geometry used and the Table 12 the material data.

Additional parameters for all Regions:

$B^2=10^{-4}$ axial buckling for both energy groups.

ν=2.43 mean number of neutrons per fission.

v_1= 3 10^7 cm s^{-1}

v_2= 3 10^5 cm s^{-1}

The Table 13 shows the delayed neutron data. Those values are not the ones reported in [4], but the ones reported in [5].

Data for Feedback model [5]:

α=3.83 10^{-11} K cm^3 conversion factor.

γ=3.034 10^{-3} $K^{0.5}$ feedback constant.

ϵ=3.204 10^{-11} Ws/fission energy conversion factor.

The initial condition is made critical by dividing the production cross sections by k_{eff}. The initial flux distribution shall be normalized such that the average power density:

$$\bar{P} = \frac{\epsilon}{V_{core}}\int_{V core}\left[\Sigma f_1(\mathbf{x},t)\Phi_1(\mathbf{x},t)+\Sigma f_2(\mathbf{x},t)\Phi_2(\mathbf{x},t)\right]\mathrm{d}V = 1\;10^{-6}\;W\;cm^{-3}$$

The initial precursor concentrations are in equilibrium with the initial critical flux distribution.

The initial temperature is $T_0 = 300$ K.

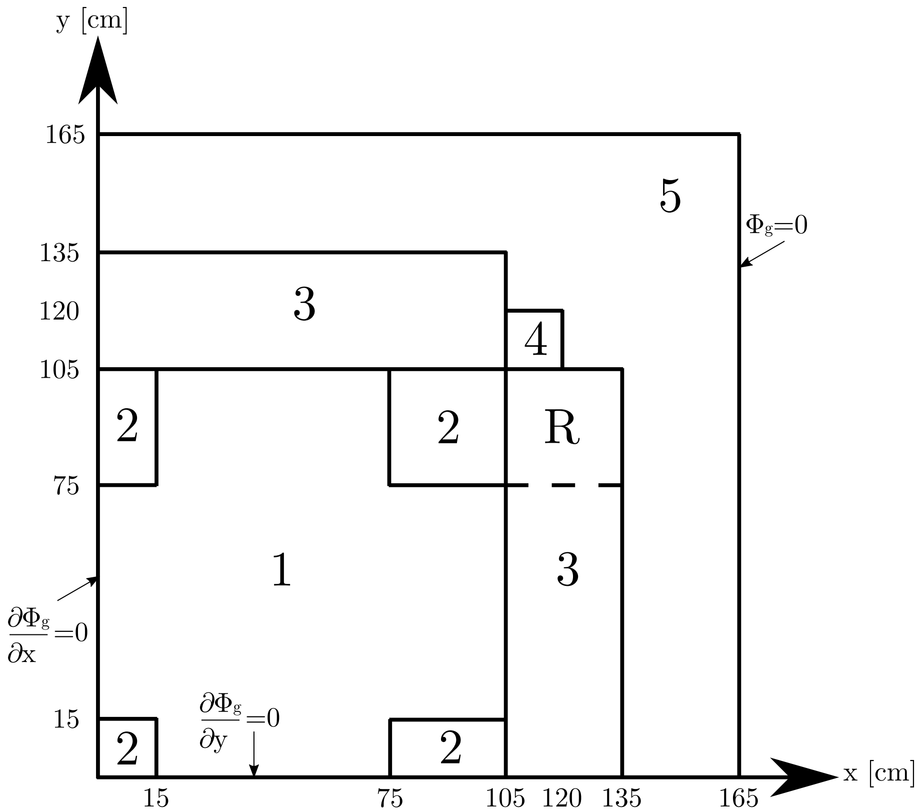

Figure 14: Geometry used to solve the 2-D LRA BWR benchmark

The initiating perturbation is such that the absorption cross section in the region R (Figure 14) changes in this way:

$$\frac{\Sigma a_2(t)}{\Sigma a_2(0)} = \begin{cases} 1-0.0606184\cdot t & ,t\le 0.2\text{s} \\ 0.8787631 & ,t>0.2\text{s}\end{cases} \qquad t = \text{time [s]}$$

Region	Material	Group i	$D_i [cm]$	$\Sigma a_i \ [cm^{-1}]$	$\nu \Sigma f_i \ [cm^{-1}]$	$\Sigma_{1 \to 2} \ [cm^{-1}]$
1	Fuel 1 with	1	1.255	0.008252	0.004602	-
	rod	2	0.211	0.1003	0.1091	0.02533
2	Fuel 1 without	1	1.268	0.007181	0.004609	-
	rod	2	0.1902	0.07047	0.08675	0.02767
3	Fuel 2 with	1	1.259	0.008002	0.004663	-
	rod	2	0.2091	0.08344	0.1021	0.02617
4	Fuel 2 without	1	1.259	0.008002	0.004663	-
	rod	2	0.2091	0.073324	0.1021	0.02617
5	Reflector	1	1.257	0.0006034	0	-
		2	0.1592	0.01911	0	0.04754

Table 12: Material data for the 2-D LRA BWR benchmark

Group	β_i	$\lambda_i \ [s^{-1}]$
1	0.0054	0.0654
2	0.001087	1.35

Table 13: Delayed neutron data for the 2-D LRA BWR benchmark

The time step is 0.001 s and there are two discretizations with quads first order (Figure 15, Figure 16).
The initial and final (t = 2 s without feedback effects) eigenvalue is shown in the Table 14.
The average power density is shown in the Figure 17.
The average temperature:

$$\bar{T} = \frac{1}{V_{core}} \int_{V core} T(\mathbf{x}, t) \mathrm{d}V$$

is shown in the Figure 18.
The maximum average power density and the time it happens is shown in the Table 15.

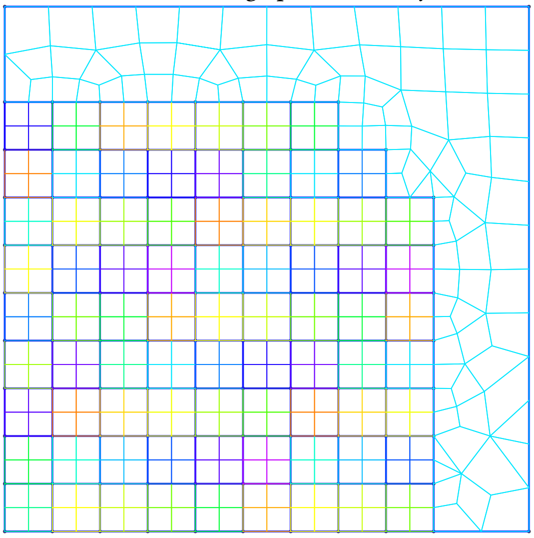

Figure 15: Coarse mesh used to solve the 2-D LRA BWR benchmark

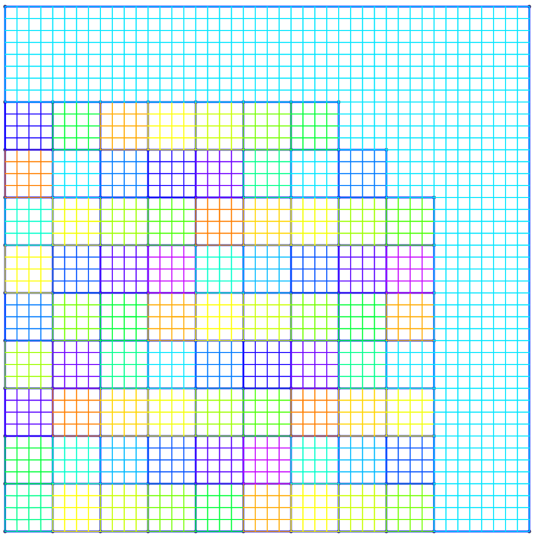

Figure 16: Fine mesh used to solve the 2-D LRA BWR benchmark

Case	Initial k_{eff}	Final k_{eff}
Coarse	0.99688	1.01663
Fine	0.99668	1.01602
Reference [4]	0.99633	1.01546

Table 14: Static calculations at 0 and 2 s

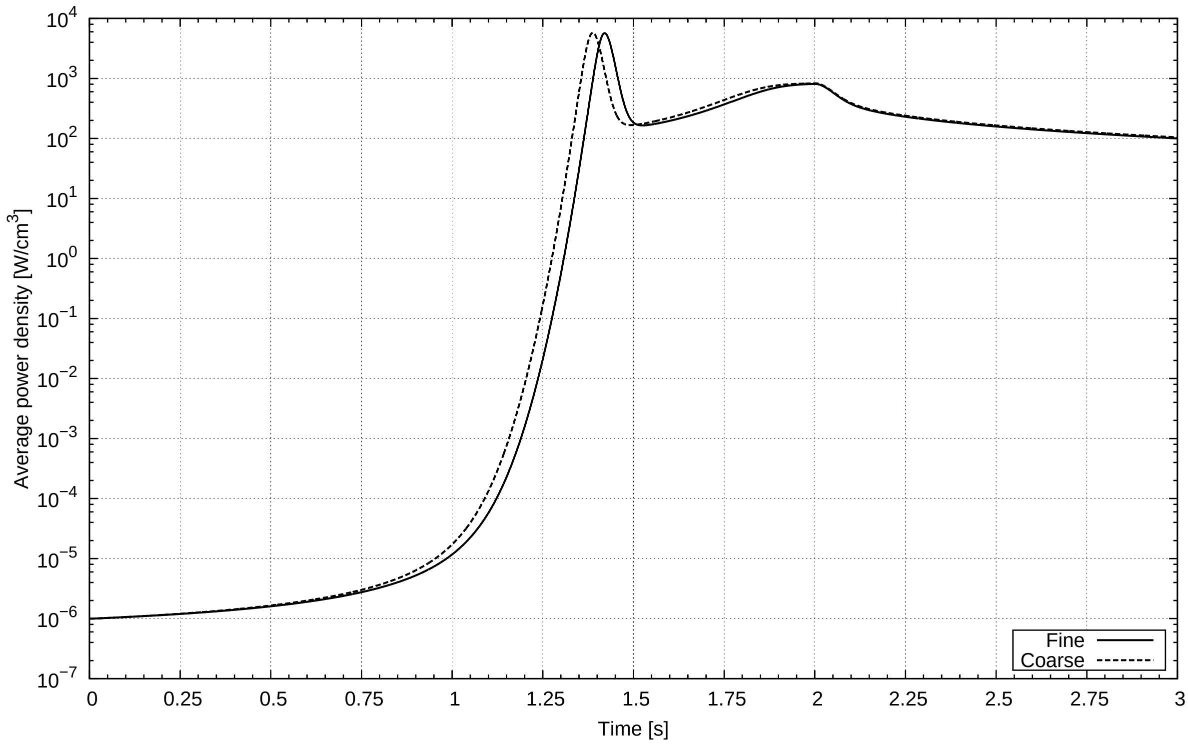

Figure 17: Average power density of 2-D LRA BWR benchmark

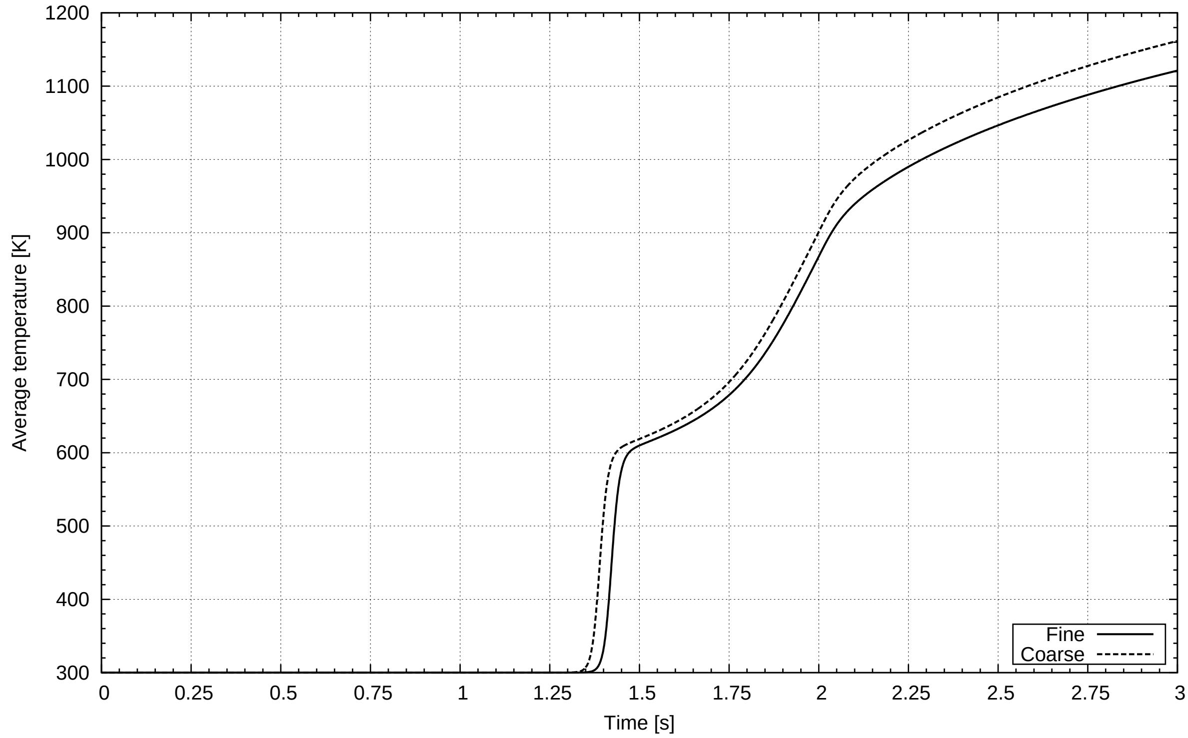

Figure 18: Average temperature of 2-D LRA BWR benchmark

Case	$\bar{P}_{max}$[W/cm³]	Time [s]
Coarse	5772	1.388
Fine	5681	1.421
Reference [4]	5734	1.421

Table 15: Maximum average power density

4 Twigl benchmark

These are two benchmarks which changes the speed of the perturbation.

There are two neutron groups and one group of delayed neutron precursors.

The Figure 19 shows the geometry used and the Table 16 shows the materials data.

The delayed neutron fraction is $\beta = 0.0075$ and the decay constant is $\lambda = 0.08$ s⁻¹.

The energy per fission is proportional to the fission cross section.

The initial power is normalized to 1.

One of the cases is a step change in the thermal absorption cross section of the material 1; whereas the second case is a ramp change in the same cross section [5].

The mesh is shown in the Figure 20. Triangles second order elements are used.

The time step is 0.005 s and it is solved with the Crank Nicolson method.

The initial eigenvalue is $k_{eff} = 0.91320$ and the final eigenvalue (after dividing the fission cross sections by the initial k_{eff}) is $k_{eff} = 1.00385$. So, the reactivity insertion from the critical state is 384 pcm.

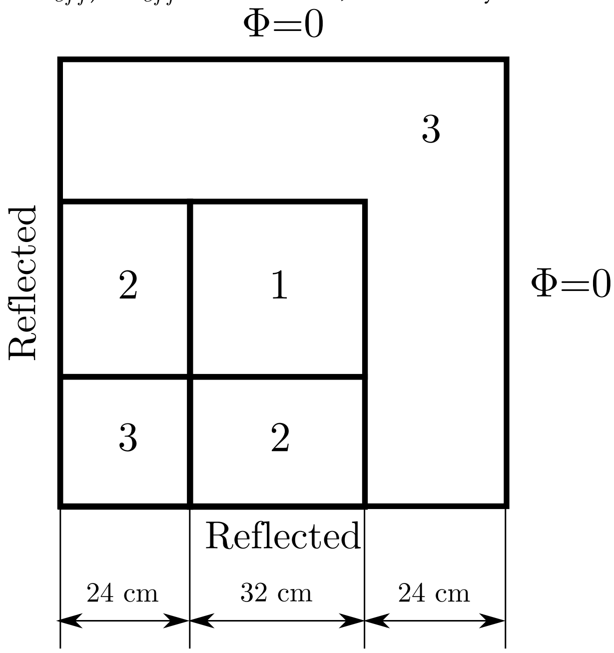

Figure 19: Geometry used to solve the Twigl benchmarks

Constant	Region 1 and 2	Region 3
D¹ [cm]	1.4	1.3
D² [cm]	0.4	0.5
Σ_a^1 [cm⁻¹]	0.01	0.008
Σ_a^2 [cm⁻¹]	0.15	0.05
$\nu\Sigma_f^1$ [cm⁻¹]	0.007	0.003
$\nu\Sigma_f^2$ [cm⁻¹]	0.2	0.06
$\Sigma^{1\rightarrow 2}$ [cm⁻¹]	0.01	0.01
χ^1 [-]	1	1
χ^2 [-]	0	0
v¹ [cm/s]	10^7	10^7
v² [cm/s]	$2\ 10^5$	$2\ 10^5$
ν^1	2.43	2.43
ν^2	2.43	2.43

Table 16: Twigl materials data

4.1 Step perturbation

The perturbation is $\Sigma_{a,1}^2(t) = \Sigma_{a,1}^2(0) \cdot 0.97666$. It means that the thermal absorption cross section of the material 1 is reduced suddenly.

The Figure 21 and the Table 17 show the power and the comparison with the reference solution [5].

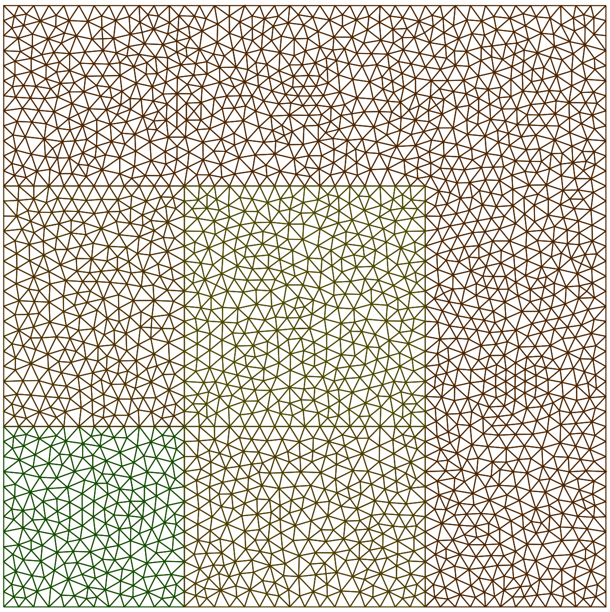

Figure 20: Mesh used in the twigl benchmark

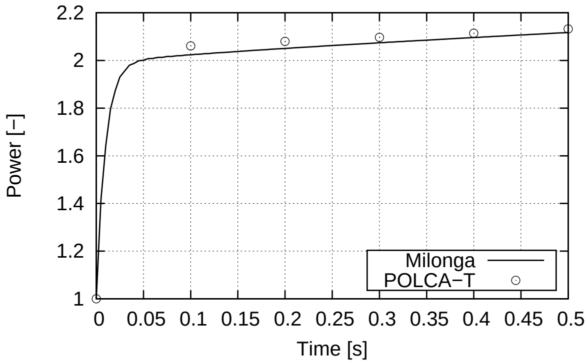

Figure 21: Relative Power

Time [s]	Milonga	POLCA-T
0	1	1
0.1	2.023	2.061
0.2	2.051	2.080
0.3	2.074	2.097
0.4	2.096	2.114
0.5	2.117	2.132

Table 17: Relative Power

4.2 Ramp perturbation

The perturbation is in the thermal absorption cross section of the material 1:

$$\Sigma^2_{a,1}(t) = \begin{cases} \Sigma^2_{a,1}(0) \cdot (1 - 0.11667 \cdot t) & , t \leq 0.2\text{s} \\ \Sigma^2_{a,1}(0) \cdot 0.97666 & , t > 0.2\text{s} \end{cases}$$

The Figure 22 and the Table 18 show the power and the comparison with the reference solution [5].

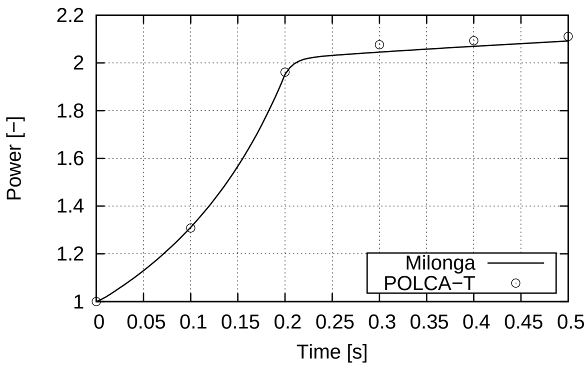

Figure 22: Relative Power

Time [s]	Milonga	POLCA-T
0	1	1
0.1	1.311	1.308
0.2	1.952	1.961
0.3	2.045	2.076
0.4	2.069	2.093
0.5	2.092	2.111

Table 18: Relative Power

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[3] *Argonne Code Center: Benchmark Problem Book, ANL-7416, Supplement 1 (1972). Identification 6: Infinite Slab Reactor Model*. 1969.

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[5] Jurij Kotchoubey. "POLCA-T Neutron Kinetics Model Benchmarking". MA thesis. AlbaNova University Center, KTH Royal Institute of Technology, Division of Nuclear Reactor Technology, 106 91 Stockholm, Sweden, 2015.