



Three single-part linear elastic benchmark problems Fino benchmark problem series

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Abstract

A benchmark of three single-part mechanical analyses solved with CAEplex using the free and open source tools Gmsh and Fino as the back-ends against a the commercial closed-source code. As the license renders this software privative, only results can be compared as output by a black-box. Even more, experts say that this "reference" software cannot export point-wise numerical data so only maximum scalar values can be compared instead of the full dependence on x, y and z of the unknowns. The comparison was made trying to keep the same number of nodes in the two codes similar while meshing the 3D CAD models with second-order tetrahedral elements. The results obtained are summarized as follows:

	SolidWorks		Fino			
Benchmark	Nodes	Displ. [mm]	σ [MPa]	Nodes	Displ. [mm]	σ [MPa]
#1	37k	0.305	88-96	36k	0.3085	89
#2	40k	0.020	42-58	40k	0.01993	56
#3	43k	0.093	165 - 241	40k	0.09307	238







Keywords Fino, benchmark, SolidWorks, linear elasticity

Revision history

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1 Introduction

This reports shows the results of solving three simple linear single-part test problem with the web-based platform CAEplex, that uses the free and open source back-ends Gmsh [1] for the mesh generation and Fino¹ for the solution of the equations that arise when employing the finite element method for discretizing the linear elastic problem. The results are compared against a reference solution of the same problems, obtained by using a commercial closed-source desktop software, and serves as a further reference for code verification and validation like the previous references [2] and [3]. The report with the reference solutions was prepared by an official technological agency of the government of the Santa Fe province in Argentina specifically for this Fino benchmark report, and is included completely in appendix D.

The reference results were obtained using SolidWorks Simulation Premium 2013.² As the license renders it privative software (because it deprives users from their basic freedoms), all we can do is compare results in a black-box sense, since we cannot know exactly what the program does as source code is not made available. Moreover, it appears that the full detailed spatial distribution of the results (i.e. displacements, derivatives, stresses, etc.) cannot be exported as numerical data as a function of x, y and z —at least that was what the experts told us—so a complete comparison of spatial dependence of the difference of results obtained with Fino on the one hand and this commercial code on the other hand cannot be done.

1.1 About CAEplex

CAEplex is a Computer Aided Engineering web-based platform which was designed from scratch to run finiteelement analysis models on the cloud. Technically speaking, it is a web front end for a number of mechanical engineering free and open source back end tools running on an UNIX server. It was developed by the Argentinian company Seamplex and launched on February 2017. CAEplex' main objective is to lower the entry barriers that traditional CAE software present to beginners, especially those working in industrial SMEs, both in *difficulty* and in *price*. Its main objective is to provide a *very easy* way to perform mechanical analysis of manufactured industrial parts.



(a) 3D CAD model rendered in FreeCAD



(b) CAEplex results screen in Chrome

Figure 1: CAEplex allows to mechanically analyze 3D parts directly from the web browser at https://www.caeplex.com.

Technically speaking, CAEplex is a web-based front-end for a number of free and open source back-ends. The two most important ones are Fino and Gmsh.

¹https://www.seamplex.com/fino

²We would like to mention this software as little as possible to avoid encouraging readers to use privative closed-source non-free software.

1.1.1 Fino & Gmsh

The problem is solved with the free³ and open source finite-element analysis tool Fino—also developed by Seamplex with a Gmsh-generated unstructured grid. More information about these programs, including documentation and downloads can be found at the following locations





(a) https://www.seamplex.com/fino

(b) http://gmsh.info/

It should be noted that, following Seamplex' principles,⁴ only free and open source software was used in preparing this report. A notable and inevitable exception is the reference report included in appendix D (elaborated by a third party, though), that uses all kind of privative software, from the operating system down to the actual calculation code.



Figure 2: The three geometries as processed with FreeCAD out of the original STEP files.

	Material	
Young's modulus	<i>E</i> =	210 GPa
Poisson coefficient	$\nu =$	0.28
Yield strength	$\sigma_{ m yield} =$	220.6 MPa

Table 1: Mechanical properties of the material used in the three problems.

2 Problems

The problems solved in this benchmark consist of three relatively simple solids presented as 3D CAD models. Even though they all have at least one plane of symmetry, the full geometry was kept and used for the spatial discretization as done in the reference report included in appendix D. The three parts are assumed to be made of the same material, namely a non-alloyed carbon steel (according to the reference software). The mechanical properties are uniform throughout the domain and their numerical values are shown on table 1.

It makes no sense to compare code-to-code finite-element solutions with different levels of spatial discretizations. So in order to obtain comparable results, the problems were solved using CAEplex by employing second-order elements (as in the reference solutions) and by trying to obtain a similar number of *nodes* rather than *elements* in both solutions. The number of nodes give directly the number of discrete unknowns of the problem (i.e. the size of the stiffness matrix). And the order of the elements give the sparsity pattern of such matrix. Therefore, these two features are a real direct measure of the computational effort needed to solve the problem. For instance, if the codes were compared by solving a problem using the same number of elements but using linear ones in the first code and quadratic ones in the other one, the comparison (and the drawn conclusions) would be definitely wrong.

2.1 Part #1

The first part is a spring that works by fixing its base and receiving a vertical load. In particular, in the problem the spring is subject to a vertical load in the negative y direction equal to 100 N, uniformly distributed in the load surface as shown in figure 3. The mesh used in the reference calculation was generated using a characteristic element length $\ell_c = 2$ mm and results in 37.0k nodes and 21.1k second-order elements, allegedly only tetrahedra. We do not know what algorithm was used to compute the reference grid.

Appendix A.4 shows the details of the meshing settings chosen to solve this problem with CAEplex, and figures 13 and 14 show the resulting unstructured grid with 36.1k nodes and 19.7k second-order tetrahedra. This grid was obtained by using the Frontal [4] algorithm for meshing both two [5] and three-dimensional [6] entities.

2.2 Part #2

The second part is a handle that should with stand a certain load perpendicular to the base (figure 4). The problem consists of fixing the base and applying a load of 1000 N in the outward direction. The reference mesh has $\ell_c = 4 \text{ mm}$ and results in 39.9k nodes and 25.7k second-order tetrahedra.

The CAEplex meshing settings are shown in appendix B.4 and the general grid can be seen in figure 25. Figure 26 shows a detail of the grid around the location where the maximum stresses are expected. The surfaces were meshed using the Frontal algorithm and the volumes using an improved Delaunay method [7], resulting in 40.0k nodes and 23.7k tetrahedra.

2.3 Part #3

The third part is a plate support, designed to be mounted on a wall with four screws. The cylindrical surfaces that hold the screws are assumed to be fixed and a vertical uniformly distributed load of 15 000 N is applied on the support surface as illustrated in figure 5. Using $\ell_c = 3$ mm, the reference mesh is composed of 43.1k nodes and 28.0k elements.

The grid used in CAEplex is described in appendix C.4. The Frontal method was used to obtain the mesh shown in figures ?? and ??. A detail of the second-order triangles generated around the location of the maximum stresses can be seen in figure ??. There are 40.1k nodes and 23.7k second-order tetrahedra.

³"Free" both as in "free speech" and in "free beer."

⁴https://www.seamplex.com/principles.html



Figure 3: Problem definition for part #1.



Figure 4



Figure 5

3 Results

As already stated in section 1, even though of course the commercial code can show pretty pictures with the threedimensional results, it cannot export the full dependence of x, y and z in any reasonable format, not even as an ASCII representation of the numerical data it displays on screen. So a full comparison between the results obtained with Fino cannot be performed.

The problems solved with CAEplex are publicly available online in the URLs listed below, and can be examined interactively by using just a web browser. There is no need to create a user account to access the project in read-only mode. The projects can be cloned and further modified to analyze variations in the mesh or in the solver settings, although a (free) user account is needed for this.

As CAEplex tracks changes in the project, the revision id of each of the three projects used throughout this work is also listed. This way, should the public project have modifications after the report is issued, the current results can be reproduced by reverting the project back to the reported revision.

Benchmark	URL	Revision
#1	https://caeplex.com/r?0a0	e27845e
#2	https://caeplex.com/r?ba7	5d5f12c
#3	https://caeplex.com/r?7bf	3d59c84

3.1 Displacements

Even though the CAEplex platform does not currently allow to see the magnitude of the displacements online in its embedded post-processing view, it can export a the detailed results along with the mesh topology in a VTK file for its further processing, for example with the free and open source tool ParaView. Figure 6 shows the scalar field of the displacements magnitude for the three problems.

In any case, the maximum displacement (which is the direct unknown of the variational formulation used by Fino, and presumably also for the commercial code) for each part obtained with both codes are summarized in the table below.

Benchmark	Fino	SolidWorks
#1	0.3085 mm	0.305 mm
#2	0.01993 mm	0.020 mm
#3	0.09307 mm	0.093 mm

It should be noted that CAEplex gives the maximum location with four significant decimal digits, while the results from the reference report in appendix D are rounded (or truncated?) down to the μ mm in an absolute way.

3.2 Stresses

In the displacement-based variational formulation, the fields of node displacements $u(\mathbf{x})$, $v(\mathbf{x})$ and $w(\mathbf{x})$ in each direction are the main unknown. That is to say, the vector that right-multiplies the stiffness matrix to obtain the loads contains the node displacements. The stresses, on the other hand, are secondary unknowns in the sense they have to be computed by first taking the spatial derivatives of each field $u(\mathbf{x})$, $v(\mathbf{x})$ and $w(\mathbf{x})$ with respect to each direction x, y and z, and then using the strain-stress constitutive relationships. This situation plus the fact that finite-element results tend to give node-based results whilst the shape-function derivatives are not continuous at the nodes renders the evaluation of stresses into a far more difficult endeavor—especially for second-order elements. Fino provides a few slightly different choices for this [8]. In general, Gauss-averaged methods avoid spurious highly-localized peaks while node-averaged methods give smoother results. See appendixes A.5, B.5 and C.5 for the actual settings used in each problem.

The spatial distribution of the Von Mises stress obtained with CAEplex can be seen in the appendices A.6, B.6 and C.6. On the other hand, the reference report is not clear about what the maximum Von Mises stress is in each case but it gives three candidate values. Even more, the units of stresses changed from SI to kilograms-force per squared centimeter. For the sake of comparison, we take the reference results as a range by converting back to MPa:

Benchmark	Fino	SolidWorks
#1	89 MPa	88–96 MPa
#2	56 MPa	42–58 MPa
#3	238 MPa	165–241 MPa



Figure 6: Displacement field (magnitude) as post-processed with ParaView out of the VTK output CAEplex provides.

4 Conclusions

Three single-part linear elastic problems were solved using a commercial code (by a technical department of the government of the Santa Fe province in Argentina) and by Fino, through the web-based front-end CAEplex. Actually, the projects can be accessed online at the short URLs shown in page 10, even without needing to create an account at caeplex.com. The CAEplex-Fino solution was obtained by trying to keep the same number of nodes in a second-order tetrahedron-based discretization of the domain.

Apparently, the commercial code cannot export the detailed spatial distribution of results so a full comparison of the solution fields cannot be done. Nevertheless, the maximum displacements—that are the primary unknowns, i.e. the solution of the finite-element discretized problem casted as a linear system of equations—coincide in two of the three cases up to the significant digits reported in the reference solution. The other case shows a 1% difference, that may be attributed to differences in the grid. For all practical purposes, the displacements coincide.

On the other hand, the stresses—which are more complex to obtain and that have far more uncertainty as they have to be computed out of the gradient fields of the displacements and evaluated at the nodes[sigmas]—also coincide in their maximum values, although the reference report gives a range of candidate values for the maximum Von Mises stress. In two of the three cases (coincidentally the two that give the same displacements with both codes), σ_{max} is located in a sharp edge (part #3) or in small-radius fillet (part #2). It is unclear how the commercial closed-source privative software computes stresses out of the displacements. In Fino, a Gauss-averaged method, that avoids unphysical high derivatives in nodes located in sharp edges, was used in these two problems. In the other problem, an average of the derivatives evaluated at the nodes was used, as this method gives smoother derivatives. In all cases, the maximum Von Mises stresses are consistent between the two finite-element codes.

References

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- [7] H. Si. Tetgen: a quality tetrahedral mesh generator and three-dimensional Delaunay triangulator. Tech. rep. 2004.
- [8] G. Theler. Evaluation of partial derivatives of pointwise node-based data over unstructured grids. Technical Note SP-WA-16-TN-9DB0. Version A. Seamplex, Nov. 23, 2016.

A CAEplex project for part #1

	CAEplex ⁵ project settings
Name	Benchmark part 1
Author	Jeremy Theler
Owner	jeremy
Туре	Linear structural analysis
Project ID	0a051cb92dd0e2e528dbf91e11facf2e
Creation date	Tue, 12 Sep 2017 15:39:43 -0400 (EDT)
Last modification	Wed, 20 Sep 2017 15:53:29 -0400 (EDT)
Revision	e27845e

A.1 Project description

Benchmark to compare CAEplex (Fino) against other FEM codes. Reference solution is max. displ = 0.305mm and max. sigma = 88-96 MPa.





Figure 7: Project URL https://caeplex.com/r?0a0

 $^{{}^{\}scriptscriptstyle 5}\!CAE plex$ is a web-based front-end for cloud-based open-source finite element analysis codes.

A.2 Geometry

Geometry file	
Name	Benchmark part 1
Format	STEP
MD5	8f449db0d8d8e6b04fde22963586a19e
Original size	0.07 Mb
Encrypted size	0.01 Mb
Length units	Millimeters

OpenCASCADE topological entitites	
Vertices	48
Edges	78
Faces	32
Solids	1
Wires	34
Shells	1
Compsolids	0
Compounds	0

Center of gravity	
x_{cog}	59.92 mm
y_{cog}	5.2459 mm
$z_{ m cog}$	0 mm

Boun	ding box
x_{\min}	0 mm
x_{\max}	130 mm
y_{\min}	0 mm
$y_{\rm max}$	30 mm
z_{\min}	0 mm
$z_{ m max}$	0.13 mm



Figure 8: Problem CAD model

A.3 Problem

	Material	
Name	Carb	on+Steel
Young modulus	E =	210 GPa
Poisson ratio	$\nu =$	0.28
Yield strength	$\sigma_{\rm yield} =$	221 MPa
Density	$\rho =$	7800 kg/m3

	BC #1: fixation
Туре	Displacement (Dirichlet)
Condition	Fixed

	Volumetric forces	
Туре		None

	BC #2: load
Туре	Load (Neumann)
Condition	Force
	$F_x = 0 \ \mathrm{N}$
	$F_y = -100 \ \mathrm{N}$
	$F_z = 0 \ \mathrm{N}$



Figure 9: Boundary conditions: vertical load surface in green



Figure 10: Boundary conditions: fixed face in magenta

Mesh A.4

Unstructured grid			Meshin	g settings
Back-end version	Back-end version Gmsh 3.0.5		Characteristic length ℓ_c	2.1 mm
Nodes	36,198		2D algorithm	Frontal
Elements	(quadratic) 29,110		3D algorithm	Frontal
Triangles	9,372		ℓ_c from	points boundaries
Tetrahedra	19,738		Optimization	basic netgen lloyd high

$\xi(r,s) = (1-r-s)$	$h_{\lambda}(r, q) = 4 \xi(r, q) r$
$h_1(r,s) = \xi(r,s) \cdot [2 \cdot \xi(r,s) - 1]$	$n_4(r,s) = 4 \cdot \zeta(r,s) \cdot r$
$h_2(r,s) = r \cdot (2 \cdot r - 1)$	$h_5(r,s) = 4 \cdot r \cdot s$
$h_3(r,s) = s \cdot (2 \cdot s - 1)$	$h_6(r,s) = 4 \cdot s \cdot \xi(r,s)$

(a) Shape functions



(b) Gauss points



Figure 11: Mathematical details of the triangular second-order isoparametric elements

$\xi(r, s, t) = 1 - r - s - t$	$h_5(r,s,t) = 4 \cdot \xi \cdot r$	Index	Weight	r	s	t
$\zeta(r, s, t) = 1$ 7 5 t $h_1(r, s, t) = \zeta [2, \zeta, 1]$	$h_6(r,s,t) = 4 \cdot r \cdot s$	1	1/4	$\frac{5-\sqrt{5}}{20}$	$\frac{5-\sqrt{5}}{20}$	$\frac{5-\sqrt{5}}{20}$
$h_1(t,s,t) = \zeta \cdot [2 \cdot \zeta - 1]$	$h_7(r,s,t) = 4 \cdot s \cdot \xi$	2	1/4	$\frac{5+3\sqrt{5}}{20}$	$\frac{5-\sqrt{5}}{20}$	$\frac{5-\sqrt{5}}{20}$
$h_2(r, s, t) = r \cdot (2 \cdot r - 1)$	$h_8(r,s,t) = 4 \cdot \xi \cdot t$	3	1/4	$\frac{5-\sqrt{5}}{20}$	$\frac{5+3\sqrt{5}}{20}$	$\frac{5-\sqrt{5}}{20}$
$h_3(r, s, t) = s \cdot (2 \cdot s - 1)$	$h_9(r, s, t) = 4 \cdot s \cdot t$	4	1/4	$\frac{5-\sqrt{5}}{20}$	$\frac{5-\sqrt{5}}{20}$	$\frac{5+3\sqrt{5}}{20}$
$h_4(r,s,t) = t \cdot (2 \cdot t - 1)$	$h_{10}(r,s,t) = 4 \cdot r \cdot t$		(b)	Gauss poin	ts	

(a) Shape functions





(c) General coordinates

(d) Real \leftrightarrow Canonical



(0, 0, 1)

5050

(0, 0.5, 0.5)

Figure 12: Mathematical details of the tetrahedral second-order isoparametric elements



Figure 14: Problem grid

A.5 Solution

	Back-end details
Back-end version	Fino v0.5.78-gcd4cd47
Formulation	Variational displacement-based linear elastic problem
Weighting	Galerkin
Problem size (unknowns)	108,594
Solver library	PETSc 3.7.5
Preconditioner	Default (GAMG)
Krylov method	Default (GMRES)
DOF ordering	Default (Node-based)
Gradient evaluation	Nodal_average

A.5.1 Stiffness matrix



Figure 15: Problem stiffness matrix (108,594 imes 108,594). Blue (red) dots represent positive (negative) values.

A.6 Results

Displacement		Von Mises stresses			
Maximum value	$ (u, v, w) _{\max} =$	0.3085 mm	Material yield strength	$\sigma_{\rm yield} =$	221 MPa
Location	x =	92.5 mm	Maximum Von Mises stress	$\sigma_{\rm max} =$	88.77 MPa
	y =	30 mm	Location	x =	49.1 mm
	z =	-38.1 mm		y =	0.128 mm
				z =	-25.9 mm
			Mean value	$\sigma_{\rm mean} =$	4.382 MPa
			Peak factor	$f_p =$	20.26

Maximum load level with respect to material yield



Figure 16: Von mises stresses and warped displacements with respect to the original geometry







Figure 18: Side warped view

B CAEplex project for part #2

	CAEplex ⁶ project settings
Name	Benchmark part 2
Author	Jeremy Theler
Owner	jeremy
Туре	Linear structural analysis
Project ID	ba7e27719a4bc8e9a89e37b59610c925
Creation date	Tue, 12 Sep 2017 15:40:12 -0400 (EDT)
Last modification	Wed, 20 Sep 2017 16:07:16 -0400 (EDT)
Revision	5d5f12c

B.1 Project description

Benchmark to compare CAEplex (Fino) against other FEM codes. Reference solution is max. displ = 0.020mm and max. sigma = 42-58 MPa.





Figure 19: Project URL https://caeplex.com/r?ba7

 $^{^{\}rm 6}\!CAE$ plex is a web-based front-end for cloud-based open-source finite element analysis codes.

B.2 Geometry

	Geometry file
Name	Benchmark part 2
Format	STEP
MD5	281e653feca87ae2103f16c6fa1e0b85
Original size	0.05 Mb
Encrypted size	0.01 Mb
Length units	Millimeters

OpenCASCADE topological entitites	
Vertices	28
Edges	46
Faces	20
Solids	1
Wires	22
Shells	1
Compsolids	0
Compounds	0

Center of gravity		
x_{cog}	0 mm	
y_{cog}	13.5306 mm	
$z_{\rm cog}$	0 mm	

Bound	ling box
x_{\min}	0 mm
x_{\max}	45 mm
y_{\min}	0 mm
$y_{\rm max}$	68 mm
z_{\min}	0 mm
$z_{\rm max}$	75 mm



Figure 20: Problem CAD model

B.3 Problem

	Material	
Name	Carb	on+Steel
Young modulus	E =	210 GPa
Poisson ratio	$\nu =$	0.28
Yield strength	$\sigma_{\rm yield} =$	221 MPa
Density	$\rho =$	7800 kg/m3

	BC #1: fixation
Туре	Displacement (Dirichlet)
Condition	Fixed

	BC #2: load
Туре	Load (Neumann)
Condition	Force
	$F_x = 0 \ \mathrm{N}$
	$F_y = 1000 \ \mathrm{N}$
	$F_z = 0 \ \mathrm{N}$

Volumetric forces

None



Туре

Figure 21: Boundary conditions: vertical load surface in green



Figure 22: Boundary conditions: fixed face in magenta

B.4 Mesh

Unstructured grid			Meshing settings
Back-end version	Gmsh 3.0.5	Characteristic	$\ell {\rm length} \ell_c $ 3.8 mm
Nodes	40,057	2D algorithm	Frontal
Elements	(quadratic) 31,775	3D algorithm	Newdelaunay
Triangles	8,004	ℓ_c from	points curvature boundaries
Tetrahedra	23,771	Optimization	basic netgen lloyd high

$\xi(r,s) = (1 - r - s)$	$h(m, c) = \int f(m, c) m$
$h_1(r,s) = \xi(r,s) \cdot [2 \cdot \xi(r,s) - 1]$	$h_4(r,s) = 4 \cdot \zeta(r,s) \cdot r$ $h_7(r,s) = 4 \cdot r \cdot s$
$h_2(r,s) = r \cdot (2 \cdot r - 1)$	$h_{5}(r,s) = 4 \cdot r \cdot s$ $h_{c}(r,s) = 4 \cdot s \cdot \xi(r,s)$
$h_3(r,s) = s \cdot (2 \cdot s - 1)$	$m_0(r, s) = 4 \cdot s \cdot \zeta(r, s)$

(a) Shape functions

Index Weight rs1 1/31/6 1/6 1/32 2/31/63 1/31/62/3

(b) Gauss points



Figure 23: Mathematical details of the triangular second-order isoparametric elements

$\xi(r, s, t) = 1 - r - s - t$	$h_5(r,s,t) = 4 \cdot \xi \cdot r$	Index	Weight	r	s	t
$\zeta(r, s, t) = 1$ 7 5 t $h_1(r, s, t) = \zeta [2, \zeta, 1]$	$h_6(r, s, t) = 4 \cdot r \cdot s$	1	1/4	$\frac{5-\sqrt{5}}{20}$	$\frac{5-\sqrt{5}}{20}$	$\frac{5-\sqrt{5}}{20}$
$h_1(t,s,t) = \zeta \cdot [2 \cdot \zeta - 1]$	$h_7(r,s,t) = 4 \cdot s \cdot \xi$	2	1/4	$\frac{5+3\sqrt{5}}{20}$	$\frac{5-\sqrt{5}}{20}$	$\frac{5-\sqrt{5}}{20}$
$h_2(r, s, t) = r \cdot (2 \cdot r - 1)$	$h_8(r,s,t) = 4 \cdot \xi \cdot t$	3	1/4	$\frac{5-\sqrt{5}}{20}$	$\frac{5+3\sqrt{5}}{20}$	$\frac{5-\sqrt{5}}{20}$
$h_3(r, s, t) = s \cdot (2 \cdot s - 1)$	$h_9(r, s, t) = 4 \cdot s \cdot t$	4	1/4	$\frac{5-\sqrt{5}}{20}$	$\frac{5-\sqrt{5}}{20}$	$\frac{5+3\sqrt{5}}{20}$
$h_4(r,s,t) = t \cdot (2 \cdot t - 1)$	$h_{10}(r,s,t) = 4 \cdot r \cdot t$		(b)	Gauss poin	ts	

(a) Shape functions





(e) Canonical coordinates

Figure 24: Mathematical details of the tetrahedral second-order isoparametric elements



Figure 26: Mesh detail

B.5 Solution

	Back-end details
Back-end version	Fino v0.5.78-gcd4cd47
Formulation	Variational displacement-based linear elastic problem
Weighting	Galerkin
Problem size (unknowns)	120,171
Solver library	PETSc 3.7.5
Preconditioner	Default (GAMG)
Krylov method	Default (GMRES)
DOF ordering	Default (Node-based)
Gradient evaluation	Gauss_average

B.5.1 Stiffness matrix



Figure 27: Problem stiffness matrix (120,171 imes 120,171). Blue (red) dots represent positive (negative) values.

B.6 Results

	Displacement		Von Mises s	tresses	
Maximum	$ (u, v, w) _{\max} =$	0.019 93 mm	Material yield strength	$\sigma_{\rm yield} =$	221 MPa
value			Maximum Von Mises stress	$\sigma_{\rm max} =$	56.19 MPa
Location	x =	7.5 mm	Location	x =	-1.08 mm
	y =	60.1 mm		y =	51.2 mm
	z =	$8.69 imes10^{-13}~\mathrm{mm}$		z =	47.4 mm
			Mean value	$\sigma_{\rm mean} =$	1.374 MPa
			Peak factor	$f_p =$	40.90

Maximum load level with respect to material yield



Figure 28: Von mises stresses









C CAEplex project for part #3

	CAEplex ⁷ project settings
Name	Benchmark part 3
Author	Jeremy Theler
Owner	jeremy
Туре	Linear structural analysis
Project ID	7bf708acd0b6196f21af91f8d63962ad
Creation date	Tue, 12 Sep 2017 15:40:18 -0400 (EDT)
Last modification	Wed, 20 Sep 2017 16:11:40 -0400 (EDT)
Revision	3d59c84

C.1 Project description

Benchmark to compare CAEplex (Fino) against other FEM codes. Reference solution is max. displ = 0.093mm and max. sigma = 165-241 MPa.





Figure 31: Project URL https://caeplex.com/r?7bf

⁷CAEplex is a web-based front-end for cloud-based open-source finite element analysis codes.

C.2 Geometry

Geometry file				
Name	Benchmark part 3			
Format	STEP			
MD5	f69bbabc1a669f47db058f6eaf379d8c			
Original size	0.09 Mb			
Encrypted size	0.01 Mb			
Length units	Millimeters			

OpenCASCADE topological entitites			
Vertices	64		
Edges	100		
Faces	38		
Solids	1		
Wires	46		
Shells	1		
Compsolids	0		
Compounds	0		

Center of gravity			
$x_{ m cog}$	35 mm		
$y_{ m cog}$	42.2743 mm		
$z_{ m cog}$	22.1226 mm		

Bounding b	ox
x_{\min}	0 mm
x_{\max}	70 mm
y_{\min}	0 mm
y_{\max}	80 mm
$z_{ m min}$	0 mm
z_{\max}	70 mm

C.3 Problem

Material				
Name Carbon+Steel				
Young modulus	E =	210 GPa		
Poisson ratio	$\nu =$	0.28		
Yield strength	$\sigma_{\rm yield} =$	221 MPa		
Density	$\rho =$	7800 kg/m3		

	BC #1: fixation
Туре	Displacement (Dirichlet)
Condition	Fixed

	BC #2: load
Туре	Load (Neumann)
Condition	Force
	$F_x = 0 \ \mathrm{N}$
	$F_y=-15000~{ m N}$
	$F_z = 0 \ \mathrm{N}$

Volumetric forces

None

Туре











Figure 34: Boundary conditions: fixed face in magenta

C.4 Mesh

Unstructured grid		Ν	Meshing settings	
Back-end version	Gmsh 3.0.5	Characteristic ler	ngth ℓ_c 2.9	9 mr
Nodes	40,126	2D algorithm	Fre	onta
Elements	(quadratic) 31,871	3D algorithm	Fre	onta
Triangles	8,152	ℓ_c from	points curvature bound	larie
Tetrahedra	23,719	Optimization	basic netgen lloyd	high

$\xi(r,s) = (1 - r - s)$ $h_1(r,s) = \xi(r,s) \cdot [2 \cdot \xi(r,s) - 1]$ $h_2(r,s) = r \cdot (2 \cdot r - 1)$	$h_4(r,s) = 4 \cdot \xi(r,s) \cdot r$ $h_5(r,s) = 4 \cdot r \cdot s$ $h_6(r,s) = 4 \cdot s \cdot \xi(r,s)$
$h_3(r,s) = s \cdot (2 \cdot s - 1)$	$n_6(\tau, s) = 4 \cdot s \cdot \zeta(\tau, s)$

(a) Shape functions

Index Weight rs1 1/31/6 1/6 2 1/32/31/6 3 1/31/62/3

(b) Gauss points



Figure 35: Mathematical details of the triangular second-order isoparametric elements

$\xi(r, s, t) = 1 - r - s - t$	$h_5(r,s,t) = 4 \cdot \xi \cdot r$	Index	Weight	r	s	t
$\zeta(r, s, t) = 1$ 7 5 t $h_1(r, s, t) = \zeta [2, \zeta, 1]$	$h_6(r,s,t) = 4 \cdot r \cdot s$	1	1/4	$\frac{5-\sqrt{5}}{20}$	$\frac{5-\sqrt{5}}{20}$	$\frac{5-\sqrt{5}}{20}$
$h_1(t,s,t) = \zeta \cdot [2 \cdot \zeta - 1]$	$h_7(r,s,t) = 4 \cdot s \cdot \xi$	2	1/4	$\frac{5+3\sqrt{5}}{20}$	$\frac{5-\sqrt{5}}{20}$	$\frac{5-\sqrt{5}}{20}$
$h_2(r, s, t) = r \cdot (2 \cdot r - 1)$	$h_8(r,s,t) = 4 \cdot \xi \cdot t$	3	1/4	$\frac{5-\sqrt{5}}{20}$	$\frac{5+3\sqrt{5}}{20}$	$\frac{5-\sqrt{5}}{20}$
$h_3(r, s, t) = s \cdot (2 \cdot s - 1)$	$h_9(r, s, t) = 4 \cdot s \cdot t$	4	1/4	$\frac{5-\sqrt{5}}{20}$	$\frac{5-\sqrt{5}}{20}$	$\frac{5+3\sqrt{5}}{20}$
$h_4(r,s,t) = t \cdot (2 \cdot t - 1)$	$h_{10}(r,s,t) = 4 \cdot r \cdot t$		(b)	Gauss poin	ts	

(a) Shape functions

(c) General coordinates





(e) Canonical coordinates

Figure 36: Mathematical details of the tetrahedral second-order isoparametric elements



Figure 38: Problem grid



🕭 SEAMPLEX

C.5 Solution

	Back-end details
Back-end version	Fino v0.5.78-gcd4cd47
Formulation	Variational displacement-based linear elastic problem
Weighting	Galerkin
Problem size (unknowns)	120,378
Solver library	PETSc 3.7.5
Preconditioner	Default (GAMG)
Krylov method	Default (GMRES)
DOF ordering	Default (Node-based)
Gradient evaluation	Gauss_average

C.5.1 Stiffness matrix



Figure 40: Problem stiffness matrix (120,378 imes 120,378). Blue (red) dots represent positive (negative) values.

C.6 Results

Displacement			Von Mises stresses		
Maximum value	$ (u, v, w) _{\max} =$	0.093 07 mm	Material yield strength	$\sigma_{\rm yield} =$	221 MPa
Location	x =	61.3 mm	Maximum Von Mises stress	$\sigma_{\rm max} =$	238.1 MPa
	y =	52.6 mm	Location	x =	28 mm
	z =	69.9 mm		y =	13.2 mm
				z =	10.3 mm
			Mean value	$\sigma_{\rm mean} =$	25.64 MPa
			Peak factor	$f_n =$	9.29

Maximum load level with respect to material yield

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Figure 41: Von mises stresses and warped displacements with respect to the original geometry



rk part 3 caeplex.com/r?7bf caeplex.com/r?7bf caeplex.com/r?7bf http

Figure 43: Detailed view of the maximum stress location

D Reference report

The reference report OT464-2017 was prepared by the department of Diseño y Simulación (translated as Design and Simulation)⁸ at Dirección General de Asistencia Técnica (translated as General Direction of Technical Assistance), which is an official agency of the government of the Santa Fe province in Argentina. This work was done specifically for validating the code Fino against a commercial code. The report authors are

- Mauro Zocco
- Ignacio Ferracuti
- Gerardo Bellotti

Unfortunately, the report is only available in Spanish, and it has some parts that may be either

- 1. improved, or
- 2. inaccurate, or
- 3. wrong.

The master report SP-FI-17-BM-9AD is released under the terms of the Creative Commons Attribution-NoDerivatives 4.0 International license, and so this appendix also inherits the condition that no derivative work shall be obtained from this report. Actually, this condition was imposed on the reference report OT464-2017, so the master report needed to be released under a compatible license.

We would like to thank Gerardo Bellotti for making the arrangements for performing this study free of charge for Seamplex.

⁸Read https://www.seamplex.com/blog/say-modeling-not-simulation.html for a glimpse of what we think about the term "simulation" applied to solving deterministic equations at Seamplex.



















